Local 3-approximation algorithms for weighted dominating set and vertex cover in quasi unit-disk graphs

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LOCALGOS
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Introduction

Local algorithms: output at each node depends only on the constant-radius neighbourhood of the node
(Linial 1992, Naor and Stockmeyer 1995)

Assumptions:

- Unit-disk graphs
- Each node knows its coordinates

Problems:

- Dominating set
- Vertex cover
Prior work

Dominating set:

- 15-approximation (Urrutia 2007)
- 5-approximation (Czyzowicz et al. 2008)
- $(1 + \epsilon)$-approximation (Wiese and Kranakis 2007)

Vertex cover:

- 12-approximation trivial
- $(1 + \epsilon)$-approximation (Wiese and Kranakis 2008)
Our contributions

Simple local algorithm

3-approximation

Small local horizon (locality distance):

- Present algorithm: $r = 83$
- Wiese and Kranakis (2007): $r = 46814$ for 3-approximation

Quasi unit-disk graphs

Weighted versions
Dominating set

Input — assumed to be a unit disk graph
Dominating set

An optimal solution
Dominating set: local algorithm
Divide the plane into $2 \times 4$ rectangles
Dominating set: local algorithm

3-colour the rectangles
Dominating set: local algorithm

For each rectangle...
For each rectangle construct an extended rectangle
Extended rectangles are non-intersecting for each colour
Dominating set: local algorithm

Extended rectangles are non-intersecting for each colour
Dominating set: local algorithm

Extended rectangles are non-intersecting for each colour
Dominating set: local algorithm

For each extended rectangle...
For each extended rectangle, form a subproblem...
Dominating set: local algorithm

\[ \ldots \text{and solve the subproblem optimally} \]
Dominating set: local algorithm

Only inside needs to be dominated
Dominating set: local algorithm

Repeat for each rectangle
Dominating set: local algorithm

Repeat for each rectangle
Dominating set: local algorithm

Repeat for each rectangle
Dominating set: local algorithm

Union of local solutions
Dominating set: feasibility

Each node is dominated in at least one subproblem
Dominating set: approximation ratio

OPT is a feasible solution to each subproblem
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Dominating set: approximation ratio

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Dominating set: approximation ratio

Factor 3 approximation from 3-colouring
Vertex cover

The same basic approach applies here as well
Local horizon: worst case

Consider a shortest path within an extended rectangle
Local horizon: worst case

Pick even nodes — distance between any pair $> 1$
Local horizon: worst case

Place disks of radius $1/2$ on even nodes — non-intersecting
Local horizon: worst case

Area bound: at most 42 such disks $\Rightarrow$ at most 83 nodes
Local horizon: average case
Local horizon: average case
Conclusions

Local 3-approximation algorithm for dominating set and vertex cover

Assumptions: (quasi) unit-disk graphs, coordinates known

Unweighted case: local and poly-time

Weighted case: local — but not necessarily poly-time!

▶ Other complexity measures for local algorithms besides the local horizon?

Challenge: apply the same idea to other problems!

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