Strategies to Mitigate Off-Nominal Events in Super Dense Operations

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Given the concept of operations defining the nominal conditions for Super Dense Operations (SDO) in the Next Generation Air Transportation System (NextGen), we investigate strategies to mitigate off-nominal events that disrupt SDO. We are interested in short-duration off-nominal events in which SDO is temporarily interrupted, and through mitigation strategies, the system can eventually return to nominal SDO conditions. In this paper, we study a range of off-nominal events that can be mitigated through algorithmic traffic flow management approaches.

Abbreviations

<table>
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<tr>
<th>Abbreviation</th>
<th>Meaning</th>
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<tr>
<td>4DT</td>
<td>4 Dimensional Trajectory</td>
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<tr>
<td>AAR</td>
<td>Airport Arrival Rate</td>
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<td>ADR</td>
<td>Airport Departure Rate</td>
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<td>ATM</td>
<td>Air Traffic Management</td>
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<td>ATSP</td>
<td>Air Traffic Service Provider</td>
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<td>CPM</td>
<td>Contingency Path Map</td>
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<td>CSPM</td>
<td>Contingency Shortest Path Map</td>
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<td>DAC</td>
<td>Dynamic Airspace Configuration</td>
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<td>DST</td>
<td>Decision Support Tool</td>
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<td>FMS</td>
<td>Flight Management System</td>
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<td>MIT</td>
<td>Miles In Trail</td>
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<td>NAS</td>
<td>National Airspace System</td>
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<td>NextGen</td>
<td>Next Generation Air Transportation System</td>
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<td>OPM</td>
<td>Optimal Path Map</td>
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<td>RNP</td>
<td>Required Navigation Performance</td>
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<td>RTA</td>
<td>Required Time of Arrival</td>
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<td>SDO</td>
<td>Super Dense Operations</td>
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<td>TFM</td>
<td>Traffic Flow Management</td>
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I. Introduction

Super Dense Operations (SDO) is an Air Traffic Management (ATM) operational concept developed for safe and efficient travel of aircraft in the transition airspace around a metropolis for the Next Generation Air Transportation System (NextGen) [JPDO06, JPDO07]. The goal of SDO is to support the highest possible throughput into and out of the metropolis while taking into consideration other criteria such as enabling fuel and time-efficient arrival and departure profiles, accommodating customer priorities and constraints, and ensuring compliance with safety (with respect to aircraft-to-aircraft separations, aircraft-to-hazardous-weather separation, and controller workload) and environmental constraints.

Prior work has defined the operational concept for SDO [KMP07a, KSS07, SSK07] and algorithmic solutions to capacity estimation for SDO [KMP07b, KPM08, KZK09], routing around convective weather hazards in SDO airspace [KPP07, PKM08], human factors issues related to the SDO design [SSE08, SSE09, SAS09], and strategic Traffic Flow Management (TFM) to set up SDO in the transition airspace [JK08]. However, in order for SDO to mature in terms of technology readiness level, the operational concept and related algorithms must be adapted to

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consider both nominal as well as off-nominal conditions. The purpose of this paper is to address off-nominal
conditions, investigating a variety of strategies to mitigate off-nominal events in SDO.

This paper is organized as follows. First, we provide background information on SDO. We then describe the
general problem statement, as well as several specific instances of algorithmic problem statements for mitigating
off-nominal conditions. Finally, we give our conclusions and references for the paper.

II. Background

In this section, we provide background material related to this effort.

A. Nominal Conditions for SDO

SDO requires that the surface/terminal domain, transition airspace domain, en route domain and their supporting
Decision Support Tools (DSTs) work together to establish SDO conditions in the National Airspace System (NAS).

Surface/Terminal DSTs control take-off times to ensure that departures meet SDO requirements, manage surface
traffic so that SDO arrivals are unimpeded by surface constraints, set Airport Arrival Rate (AAR), Airport Departure
Rate (ADR), and runway configuration changes within the metroplex with sufficient look-ahead to set up SDO in en
route and transition airspace, and integrate environmental constraints into SDO terminal routing.

Transition Airspace DSTs estimate the capacity of SDO airspace

and feed the estimates to strategic TFM algorithms. The strategic algo-

rithms set the demand into SDO transition airspace to a level

compatible with the capacity of SDO airspace, determine the mode

of operations for route structures

using either unstructured or struc-
tured routing approaches (Figure 1),

and adjust Dynamic Airspace Con-

figuration (DAC) boundaries so that

the flexible routing structures are

within controller workload limits. In

addition, DSTs allocate the number

and location of arrival and departure fixes depending on arrival versus departure demand levels, and specify the
Required Navigation Performance (RNP) requirements for routing structures and fixes based on the size of gaps
between hazardous weather constraints and aircraft navigation characteristics of the traffic demand.

En route domain DSTs manage take-off times for aircraft participating in SDO operations, allocate routing to
the SDO transition airspace boundary to meet RNP requirements, and assign en route delays such that just enough
aircraft show up at or below the capacity estimate for each quadrant of SDO airspace.

While only briefly stated here, these conditions define the nominal SDO conditions for this paper. Further
details on these components are addressed in the literature.

B. Definition of Terms for Operating Conditions

In general, we define the following operating conditions for SDO:

• **Nominal Condition.** All elements of the system are operating as designed, and operational and environmental
  factors are as planned and as forecast. Runway configurations are in service as planned. Weather phenomena
  occur as forecast with minimal forecast errors. Nominal conditions include time periods of low as well as high
  traffic loads, and include typical values of system input.

• **Off-Nominal Condition.** All elements of the system are operating as designed, but operational or
  environmental factors are not as planned or as forecast. Weather phenomena occur outside the forecast
  parameters (time, intensity, position or coverage).

• **Emergency Condition.** Most elements of the system are operating as designed, but one or more of them are in
  a condition that requires special handling. These states include airborne circumstances that are time-critical to
  safety, such as an aircraft mechanical problem or a passenger medical situation resulting in the declaration of an
  emergency.

• **Failure Condition.** One or more elements of the system cease to perform as designed, planned, or expected,
  resulting in significant impact to one or more elements of the system.
The mitigation strategy is defined as follows:

- **Mitigation Strategy.** A strategy by which the SDO system adapts to an off-nominal, emergency, or failure condition and returns in a safe and efficient manner back to nominal conditions.

The foundation of our mitigation strategies is to maintain two types of plans:

- **Nominal Plans** – A nominal plan for the controller and a nominal plan for each pilot is always maintained in the SDO plan.
- **Contingency Plans** – Pre-computing a series of contingency plans for the controller and contingency plans for each pilot establishes mitigation strategies for off-nominal, emergency, and failure conditions.

The **nominal plan for the controller** is a valid solution to the SDO route planning problem as described in the SDO operational concept and previous research (we do not perform new work in this area). For example, a valid nominal plan for the controller responsible for SDO arrival traffic is to use tree-based route planning (Figure 1(a)) for the next 30 minutes, with RNP requirements placed on the tree branches based on the expected demand and how it is spatially and temporally distributed (located at SDO transition airspace arrival gates, metered, and sequenced) and handed off as appropriate from en route controllers.

The **nominal plan for a pilot** describes the cleared 4D trajectory (4DT) and other information about the role of the aircraft in the nominal plan of the controller. For example, the nominal plan for a pilot identifies the 4DT within a tree-based routing solution, RNP requirements, merge points, Required Times of Arrival (RTA) (if any), and other information pertaining to execution of the SDO plan.

The **contingency plans for a controller** is based on an assumed off-nominal, emergency, or failure condition. For instance, if a metering fix becomes unavailable for an unspecified period of time, then the controller must have a plan to relocate all aircraft within a flow or along a branch of an SDO tree-based route planning solution to either an alternative airport, to another metering fix location, or to temporarily place aircraft in holding patterns.

The **contingency plans for a pilot** are based on the pilot’s role in the assumed off-nominal, emergency, or failure conditions and the controller’s contingency plan solution. Each pilot is assigned a different role in the contingency plan solution based on the pilot’s current state (location, altitude, RNP, weather constraints, etc.). For instance, if an aircraft declares a state of emergency, the aircraft is taken off of the nominal SDO routing solution and placed on a feasible weather avoidance route directly to a runway, alternative airport, or metering fix to expedite landing of the aircraft.

While we recognize that computing comprehensive plans for controllers and pilots may constitute a huge number of contingency plans, part of our research objective is to develop approaches to handling contingencies, the discretization of the set of mitigation strategies, and the complexities of searching for contingency plans. One research issue that we investigate is the possibility that a single mitigation strategy can be used to address multiple off-nominal, emergency, and failure conditions. Thus, we seek a compact set of mitigation strategies to address the largest possible set of potential pre-conceived failures.

III. Problem Statements

The most general problem statement addresses the planning for off-nominal conditions, even though the specific off-nominal, emergency, or failure condition is not known. The TFM system begins in a nominal SDO condition, defined by a set of weather hazards, SDO routes, RNP requirements, and related parameters. In general, an off-nominal event occurs that forces the system to transition from nominal SDO conditions to off-nominal, emergency, or failure conditions. Through a mitigation strategy, the system returns to nominal SDO conditions.

This problem statement is quite general, so it is useful to consider specific scenarios defined by off-nominal, emergency, and failure conditions in order to develop our algorithms. Generally, the scenarios can be classified into 7 categories as follows.

A. Failure Modes that Require an Aircraft Diversion

Due to mechanical failure, medical emergency, or other reasons, the SDO operations may be temporarily disrupted to accommodate an aircraft diversion. The difficulty of accommodating the disruption depends on where within the SDO tree structure the aircraft is that requires the diversion:

- **Interior of an SDO tree structure** – The diversion may be for an aircraft that is “interior” to the SDO tree structure, with other branches of the tree trapping it. (It may be on a flow or in a holding pattern in the interior of the tree.) In this case, accommodation will require that the diverted aircraft crosses other branches of the SDO tree structure, thereby impacting several other aircraft currently under SDO.
• Boundary of an SDO tree structure – The diversion may be for an aircraft that is on the “boundary” of the SDO tree structure. In this case, accommodation may be as simple as utilizing airspace that is just outside the tree’s extent, causing minimal disruption to other aircraft in the SDO tree.

• Inside an SDO holding pattern – The diversion may be for an aircraft that is inside a holding pattern. In this case, accommodation is similar to the 2 cases covered above, depending on where the holding pattern is, i.e., interior of or on the boundary of a tree structure.

The diverting aircraft has highest priority and the controller handles the other impacted traffic accordingly. In some cases, the diverting aircraft may need to change altitude or directly descend to land at the nearest airport suitable for the aircraft type.

B. Failure Modes that Address an Emergency Landing
Due to mechanical failure, medical emergency, or other reasons, the SDO operations may be temporarily disrupted to accommodate an aircraft that requires an immediate emergency landing at an airport. The aircraft with the emergency has highest priority for landing using the safest, most direct route, while all other traffic must be safely handled with secondary priority.

• Landing at the destination airport – The aircraft is safely directed, with highest priority, to the destination airport.

• Landing at a diversion airport – The aircraft is directed out of the normal route to a nearby alternate airport. In this case, the aircraft goes directly to a specific runway.

In both these cases, the emergency aircraft may be starting from a position interior to the SDO tree structure or on the exterior of the SDO tree structure, as covered in the aircraft diversion case above.

C. Failure Modes that Involve a Non-Cooperative Aircraft
A non-cooperative aircraft may cross arbitrarily over originally planned traffic flows, thereby requiring operations to modify the SDO flows in order to accommodate the non-cooperative aircraft. Because such an aircraft usually causes empty slots in the planned traffic flows, the SDO operations must handle such conditions in a manner so that all of the other affected traffic is rerouted safely around the non-cooperative aircraft and the system performance is minimally impacted.

D. Failure Modes that have a Temporary Loss of Communication
Due to technology failure or pilots’ mistakes, the SDO operations may be temporarily disrupted to accommodate an aircraft that loses communication with the ground and/or with other aircraft. The aircraft with communication failure is given the highest priority, while traffic that potentially conflicts with it is moved out of its path. In such cases, the SDO operations are similar to the ones used in the failure modes involving non-cooperative aircraft.

E. Failure Modes that have Human Errors in Communication
Due to data entry or communication errors between pilots and controllers, the SDO operations may be temporarily disrupted to accommodate aircraft that fly the “wrong” routes or “miss” a turn point in a route. In such cases, the incorrectly routed aircraft may be assigned the highest priority, while potentially conflicting traffic is moved off of its nominal routes in order to accommodate the high-priority aircraft. The SDO operations handle such situations by addressing the source of the communication error in the safest and most expeditious manner, rerouting traffic in order that, hopefully, the impact to the system performance is minimized.

F. Failure Modes that Temporarily Eliminate an Airspace Resource
Due to the temporary unavailability of an airspace resource, the SDO operations may be temporarily disrupted to accommodate the traffic that originally had planned to use the unavailable resources. Such traffic is re-directed to use safe alternative routes, which are pre-computed so that the system performance is minimally affected. All of the other impacted traffic is safely handled accordingly (e.g., directed into holding patterns), as necessary.

• Unavailable route legs – all aircraft that had planned to use the unavailable route legs are rerouted to alternative routes.

• Unavailable metering fix locations – all aircraft that had planned to use the metering fix location are rerouted to other metering fix locations that are available.

Most often, unavailable airspace resources are due to hazardous weather constraints.

G. Failure Modes that Temporarily Eliminate an Airport Resource
The SDO operations may be temporarily disrupted to accommodate the aircraft that originally planned to use an airport resource that becomes unavailable, such as an icy runway or a closed airport, etc. Such aircraft are redirected
to use safe alternative routes to land at nearby airports or are directed into a go-around or into a holding pattern to wait for the unavailable airport resources to be available again.

Off-nominal scenarios are further categorized into two types: Type I scenarios are caused by aircraft in emergency situations, and Type II scenarios are usually caused by resource unavailability, e.g., due to hazardous weather constraints or closed runways. In the list of scenarios above, A-E are of Type I, while F and G are of Type II. We develop two groups of contingency plans to address scenarios of Types I and II.

For scenarios of Type I, the contingency plans provide revised trajectory information for the set of aircraft that may potentially be affected by the emergency so that they can expeditiously be rerouted safely. The contingency plans may not make substantial changes to the trajectories in the original plan, since, in most cases, the preferred option is to use holding patterns or make speed adjustments to accommodate the emergency.

Figure 2(a) illustrates an example in which the originally planned SDO tree-based routing structure has routes 1, 2, and 3 merge into one route. In Figure 2(b) the example scenario indicates a non-cooperative aircraft whose projected trajectory crosses the planned traffic flows. In this case, the aircraft that had planned to use routes 1 and 2 are now directed into holding patterns (located in the purple airspace) to wait for the non-cooperative aircraft to pass. For aircraft originally planning to use route 3, the preferred contingency plan is to stay in route and slow down, since the non-cooperative aircraft is expected to cross route 3 in the very near future, and there is adequate distance before the crossing point in order to make a speed adjustment to safely avoid the non-cooperative aircraft.

For scenarios of Type II, the contingency plan provides information for the set of aircraft that originally had planned to use the resource that has become unavailable. In this case, while directing aircraft into holding patterns to wait for the resource to be available again is often sufficient to address the event, in many scenarios, the contingency plan computes substantially different routes for the affected aircraft.

In some specific cases, such as in transition airspace or in airspace with very high traffic loads, the originally planned routes may be densely packed. Therefore, rerouting a group of aircraft to avoid resource unavailability often

Figure 2. Examples of two types of contingency plans.

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In some specific cases, such as in transition airspace or in airspace with very high traffic loads, the originally planned routes may be densely packed. Therefore, rerouting a group of aircraft to avoid resource unavailability often
results in new crossing points between routes, which then requires more operations in the other affected routes, such as possible rerouting, holding patterns, and speed adjustments.

As shown in Figure 2(c), the unforecast weather constraint \( C_1 \) makes route 3 unavailable. Therefore, the aircraft in route 3 are now directed to merge into route 2 earlier than originally planned in order to avoid \( C_1 \), as shown in Figure 2(d). In this case, the aircraft using route 2 may also need to be adjusted accordingly, using speed adjustment or rerouting, to accommodate the new incoming aircraft from route 3.

An important aspect of addressing these failure modes is that after the contingency plan is implemented and the event has passed, the TFM system should transition back to nominal conditions.

IV. Algorithmic Approaches to Mitigate Failure Modes

Our goal is to have a set of algorithmic tools that allow us to address a wide variety of off-nominal, emergency, and failure conditions, without knowing the exact cause for any particular event. We discuss here two main approaches: (1) methods to define and formalize the concept of “operational flexibility” in routing structures, in order to evaluate and analyze plans from the point of view of robustness to unexpected scenarios; and, (2) a mathematical model that explicitly accounts for off-nominal events in making the original nominal plan, thereby allowing transition to pre-specified classes of contingency plans, encoded in the concept of “contingency path maps”.

A. Operational Flexibility in Routing Structures

Operational flexibility in routing structures requires that an airspace be specified in which off-nominal routing solutions can be located. We propose four computable metrics to quantify the degree to which there exists operational flexibility for routing structures within constrained airspaces. The goal is to quantify for a given route network \( N \) and a given airspace domain \( D \) (the “region of interest”), the degree to which \( N \), or any point, path, or subnetwork of \( N \), exhibits operational flexibility within \( D \), allowing for the possibility of off-nominal events that may require rerouting of portions of \( N \) within \( D \).

Let \( X \) be a connected subnetwork of \( N \). Thus, \( X \) could be a single point on the network \( N \), a subpath within \( N \), a subtree within \( N \), or any other connected subnetwork of \( N \). Our goal is to define a meaningful and useful notion of the operational flexibility \( f(X) \) of \( X \) within \( D \). We say that the metric is additive if, for any two disjoint subnetworks \( X \) and \( X' \) of \( N \), the following holds: \( f(X \cup X') = f(X) + f(X') \).

For each metric, we discuss how it has a corresponding local and global version. A local operational flexibility metric measures flexibility that exists “close” to the original subnetwork \( X \), according to some meaningful notion of “close”. Local operational flexibility allows for relatively minor adjustments to the SDO routing structures (local adjustment of tree branches or merge points or temporary use of holding) without the need to relocate traffic to different destination locations or to define a new topology for routes. Geometric locality is quantified by a locality parameter \( R_l \) which specifies the maximum distance a reroute is to be from the nominal route. In contrast, a global metric measures flexibility that allows for reroutings that are not especially “close” to \( X \), but may require reroutes that are at a considerable (geometric) distance from \( X \), more than \( R_l \) but less than a parameter \( R_g \), or that are “far” from \( X \) in terms of topology (e.g., “homotopy type”) or in terms of revised destination locations. We discuss these concepts in more detail below. A simple example of local operational flexibility for a merge tree \( X \) in the vicinity of ORD is illustrated in Figure 3.

![Local operational flexibility provides airspace for local reroutings, local re-design of tree-based or tube-based routing structures, or holding patterns.](image)

**Figure 3.** Local operational flexibility airspace (shaded) is defined to be close to the nominal route structure (an arrival merge tree, in this example).
Examples of local operational flexibility include the following:

- When the severe weather impact does not represent large constraints, nominal SDO routing structures can be adjusted locally (e.g., by shifting the affected routes slightly, by distance at most $R_l$) within the local operational flexibility airspace, without changing the topology of the routes, in order to mitigate problems associated with weather forecast errors.
- When aircraft need to yield to another aircraft in an emergency or wait for currently unavailable airspace resources, they can utilize holding patterns which can be invoked within the local operational flexibility airspace. The topology of the routing structure is maintained, while the affected aircraft are put in holding patterns adjacent to the routing structure, for instance, as is illustrated in Figure 3.

Some off-nominal events require adjustments to the route structure that go outside the local operational flexibility airspace. An example of global operational flexibility is when the off-nominal impact weather constraints occupy a significantly large portion of the airspace, causing the nominal routing structures to become infeasible to such an extent that the routing structures must be significantly redefined for SDO. While the new routing structures may require that one or more routes are perturbed by more than distance $R_l$, and they may differ topologically from the nominal structures, the new structures should lie within the global operational flexibility airspace with respect to the nominal structure. In particular, the new routing structures should lie within distance $R_g$ of the nominal structure. This is in order to minimize the overall impact of the new weather constraint. Our distinction between “local” and “global” operational flexibility airspace is, in some sense, rather arbitrary – we are effectively distinguishing between operational flexibility that is “close” (within distance $R_l$ and topologically equivalent) to the nominal structure and operational flexibility that is “not as close, but not too far” (within distance $R_l > R_g$ and potentially of a different topology or homotopy type) from the nominal structure. Rerouting structures that go beyond distance $R_g$ from the nominal are considered to be complete reroutes, outside the range of what will be considered to be “operational flexibility”. Finer classifications of degrees to which flexibility is “local”, beyond the two-tier classification here to a $k$-tier classification, are possible but may be more confusing than helpful. In all our cases that we consider, we do not allow the local or global operational flexibility to extend beyond sector boundaries. We also discuss for each metric how it can be applied to the dynamic setting in which constraints vary with time $t$, over a time horizon $[0, T]$.

**Metric 1: Unconstrained Airspace Metric (UAM)**

The simplest metric for operational flexibility within an airspace domain $D$ is the unconstrained airspace metric (UAM), obtained by subtracting from the volume of $D$ the volumes of the constraints within $D$ and the volumes of the air lanes of the network $N$ (the “utilized” portion of the airspace) that are not part of the subnetwork $X$. Here, by “volume” of an air lane we mean the volume of the airspace that lies within a prescribed lateral offset $X$ and a prescribed vertical offset (e.g., 1000 feet). Note that if we are focusing on a single flight level, volume is proportional to area.

**Local Metric:** For a given locality parameter $R_l$, the local UAM is equal to the unutilized airspace volume within $D$ that lies within Euclidean distance $R_l$ of the subnetwork $X$.

**Global Metric:** The global UAM is equal to the unutilized airspace volume within $D$ that lies within Euclidean distance $R_g$ of the subnetwork $X$.

**Dynamic Metric:** For each time slice $t$ in the horizon $[0, T]$, we can compute the static UAM. Summing over $t$ yields a straightforward dynamic metric. (The units are now in space-time, rather than purely space (volume).)

**Computation Algorithm:** It is straightforward to compute the volume of $D$ and the volumes of the constraints. Computation of the volumes of offset networks/routes can be computed using the Voronoi diagram of $N$, or, more simply, approximated as being proportional to the length of the network.

**Pros:** The UAM is simple and intuitive, and it is straightforward to calculate.

**Cons:** The UAM is an oversimplification of the concept of operational flexibility, not taking into account the geometric structure of the available airspace. There can, in fact, be an airspace with a high UAM that has little operational flexibility, and, similarly, an airspace with a low UAM that has relatively high operational flexibility.

**Metric 2: Constrained Airspace Metric (CAM)**

The constrained airspace metric (CAM) metric assesses operational flexibility in terms of the volume of airspace that is “close” to the subnetwork $X$, where closeness explicitly takes into account the structure of the weather constraints. In particular, the CAM measures volume that is proximal (within distance $R_l$ or $R_g$, depending if we speak of local or global) to a constraint-avoiding path that is topologically equivalent (in the “local” version) to the nominal route structure $X$ or topologically “similar” (in the “global” version). Specifically, the definition is based on the Voronoi diagram (medial axis) of the constrained airspace; we make the definition explicit below, in both its local version and its global version.
**Local Metric:** With local operational flexibility, the topology of the routing structure of the subnetwork $X$ remains fixed relative to the constraints, which include the weather constraints, the other portions of the subnetwork, and the sector boundaries. Refer to Figure 4.

The local CAM for a subnetwork $X$ (e.g., a single route or a tree of routes) with respect to airspace domain $D$ is defined as follows. Let $V(D)$ denote the Voronoi diagram (or medial axis) of $D$, considering the weather constraints and route structure of $N$ that is not part of $X$ to be “holes” in $D$. Let $X_V$ denote the retraction of $X$ onto the medial axis; thus, $X_V$ is a path/tree in the medial axis of $D$ homotopically equivalent to $X$. The **local CAM of $X$** is the total area (proportional to volume) of the union of medial disks centered on $X_V$. A disk $B(c,r)$ of radius $r$ centered at point $c$ is a medial disk if $c$ lies on the medial axis and $r$ is the distance from $c$ to the boundary of $D$ (i.e., sector boundaries and weather hazard obstacles). Since $c$ is on the medial axis, we know, by definition of the medial axis, that at least two distinct points on the boundary of $D$ are at distance $r$ from $c$, while the disk $B(c,r)$ lies fully within $D$ and disk $B(c,r)$ is the largest disk centered at $c$ that lies within $D$ (i.e., that avoids the constraints.)

![Image](https://example.com/image1)

(a) The multiply connected domain $D$ and the tree of routes $X$.
(b) The Voronoi diagram of $D$, $V(D)$, denoted by the dashed lines.
(c) A single route $X'$ and its retraction $X'_V$ onto $V(D)$.
(d) The local operational flexibility region (purple) of $X'$.
(e) The retraction of $X$ onto $V(D)$, denoted by the dashed lines.
(f) The local operational flexibility region (purple) of $X$.

**Figure 4. Defining the local CAM for operational flexibility in the cases of a merge tree $X$ or a jet route $X'$. (Grey dotted lines denote sector boundaries.)**

**Global Metric:** The “global operational flexibility” is intended to capture the possibility of modifying the topology (homotopy) of the routing structure relative to weather hazard constraints. In particular, we consider operational flexibility to include reroutes that change the topology of a route, going on the “other side” of a weather constraint.

![Image](https://example.com/image2)

(a) A single route $X$ and its retraction $X_V$ onto $V(D)$.
(b) The local operational flexibility region (purple) of $X$.
(c) The global operational flexibility region (purple) of $X$.

**Figure 5. Comparing local CAM (purple area in (b)) with global CAM (purple area shown in (c)) for route $X$.**

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We define a precise, computable notion of “global operational flexibility” for a single route or a tree of routes, \( X \), as follows; refer to Figure 5(c). Instead of restricting our attention to only those routes in \( D \) that are homotopically equivalent to \( X \), we define the \( \alpha \)-neighborhood, \( N_{\alpha}(X) \), of \( X \) as follows. \( N_{\alpha}(X) \) is the set of all points \( p \) within \( D \) such that there exist points \( a \) and \( b \) on \( X \), with \( d(a,p) + d(p,b) \leq (1+\alpha)d(a,b) \), where \( d \) represents geodesic distance (within \( D \)) and \( d_{\alpha} \) represents distance in the network \( X \). The global CAM of \( X \) is the total area of the union of all medial balls centered at points of the retraction of \( N_{\alpha}(X) \) onto the medial axis of \( D \). (Intuitively, \( N_{\alpha}(X) \) represents the locus of all points in the domain (constraint-free airspace) that are reachable by a detour path, departing from \( X \) at some point \( a \) and returning to \( X \) at some point \( b \).) The neighborhood \( N_{\alpha}(X) \) allows us to consider operational flexibility that is topologically non-local to \( X \), including routes that go around constraints differently than \( X \) does. Here, the parameter \( \alpha \) can be set to \( R_{\alpha} \), or to some function of \( R_{\alpha} \), the upper bound on distance from the nominal route structure. (Note that points at Euclidean distance at most \( R_{\alpha} \) are not necessarily at geodesic distance at most \( R_{\alpha} \); thus, \( \alpha \) and \( R_{\alpha} \) measure different parameters.) The global operational flexibility always contains the local operational flexibility.

**Dynamic Metric:** For each time slice \( t \) in the horizon \([0,T]\), we can compute the static CAM. This gives a naïve metric in the dynamic setting just by integrating over time. However, our view of the problem in space-time yields a means of also incorporating the traffic density in the metric. Specifically, given a set of trajectories (the demand/traffic), we measure the volume in space-time, just as we compute for the static CAM, but now subtracting the volumes of the “tubes” that correspond to the trajectories in space-time. The result is a metric that estimates the available flexibility around the trajectories, taking into account the spacing between aircraft (MIT) and the dynamically changing weather forecast.

**Computation Algorithm:** Using standard methods of computational geometry, the static CAM can be computed efficiently as follows:

1. Compute the Voronoi diagram (medial axis network) based on the weather constraints and sector boundaries; this requires time \( O(n \log n) \) for a domain and set of constraints of total complexity \( n \).
2. Compute the retraction of \( X \), \( X_{\alpha} \), onto the medial axis; this can be done in linear time (in fact, in time proportional to the complexity of the retraction path/tree).
3. The local CAM of \( X \) is the total area of the union of medial disks centered on \( X_{\alpha} \). The area can be computed by summing elemental areas associated with each arc of \( X_{\alpha} \), taking overall linear \((O(n))\) time.

The dynamic CAM is determined by computing the static CAM at each time slice of the forecast data, and then subtracting the volumes of the trajectory sweeps of the demand (or demand forecast).

**Pros:** The CAM makes an explicit attempt to estimate the volume of the relevant portion of the airspace domain, following the topology of the nominal route structure. It applies equally well to single jet routes, to trees, and to other network structures. It is relatively efficient to compute, at least in theory.

**Cons:** The CAM includes portions of the domain \( D \) that may not be considered accessible or relevant for off-nominal reroutings. Computation of Voronoi diagrams of multiply connected domains, and especially of domains in 3D, is challenging from an implementation point of view.

**Metric 3: Operationally Accessible Airspace Metric (OAAM)**

The operationally accessible airspace metric (OAAM) considers that portion of the domain \( D \) that can be utilized for off-nominal rerouting according to standard procedures for rerouting. Operational accessibility depends on the class of permitted operations for rerouting.

**Local Metric:** The local OAAM is defined to be the area of the domain that is operationally accessible with a maximum detour from nominal that is determined by \( R_{\alpha} \), the locality parameter.

**Global Metric:** The global OAAM is defined similarly to the local OAAM, except that the maximum detour is determined by the parameter \( R_{\alpha} \).

**Dynamic Metric:** For each time slice \( t \) in the horizon \([0,T]\), we compute the static OAAM. However, this approach does not explicitly take into account the variation in time. Much of the operationally accessible portion of the domain may not be accessible when viewed in space-time. Thus, a more accurate approach to determining the degree to which there is operational flexibility in the vicinity of the nominal route structure is to consider the standard reroutes as a family of trajectories in space-time, and then to compute the accessible volume within the space-time domain directly.

**Computation Algorithm:** The algorithm depends on the specific rerouting method. For example, consider the class of reroutes that divert a flight off its nominal route by a left/right turn of 45 degrees, followed by a right/left turn of 45 degrees to a parallel route to nominal, with a similar maneuver to merge the flight back to nominal. In the static OAAM computation, we can use configuration space approaches from motion planning methods in computational geometry to determine the operationally accessible region.

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Computation of the dynamic OAAM becomes more complex, because reroutes now become polygonal tubes in space-time, and computing the union of all allowed reroutes becomes a higher dimensional configuration space problem. One possible practical approach is to consider a simple approximation in which we discretize the space-time domain into “voxels”, and compute the number of voxels that lie within the accessibility region. This can serve as a statistic to estimate the operational flexibility metric.

**Pros:** The OAAM metric explicitly models a family of possible reroutes from the nominal route structure; thus, it may more accurately reflect the operational flexibility. It can explicitly model the dynamic setting as well, within the space-time domain.

**Cons:** The OAAM requires specific knowledge of rerouting methodology and current practice; thus, it may need to be revised as other procedures are adopted. Further, the computation of the OAAM, particularly in the dynamic setting, is not particularly simple and would likely require approximations to make it practical.

**Metric 4: Maxflow Metric (MM)**

The maxflow metric (MM) for operational flexibility is defined in terms of capacity estimation in the vicinity of the subnetwork $X$. In particular, we propose to use maxflow-mincut based methods to estimate maximum throughput within a neighborhood of $X$.

**Local Metric:** The local MM is defined to be the throughput capacity, determined by a mincut computation, within the $R_l$-radius offset of $X$. See Figure 6. Here, we compute the number of lanes available within the yellow offset region of $X$. An alternative notion of “local” MM counts additional air lane capacity only for air lanes that are homotopically equivalent to $X$. In Figure 6, there are no additional air lanes (other than the nominal air lane $X$) that follow the same topology (homotopy) as $X$; the additional air lanes (in blue) are topologically distinct – one passes north of a red constraint, and one passes south of two red constraints.

**Figure 6. Defining the MM metric.**

**Global Metric:** The global MM is defined similarly to the local MM, except that we use the (larger) offset parameter $R_g$, and, optionally, also allow the homotopy of the air lanes to be distinct from $X$ (as in Figure 6).

**Dynamic Metric:** For each time slice $t$ in the horizon $[0,T]$, we compute the static MM, and then sum over time slices. However, a more accurate estimate of dynamic MM is to do the computation in the space-time domain, using the “motion graph” approach [AMP10].

**Computation Algorithm:** The algorithm begins by computing the offset of $X$. This determines the restricted domain within which we run the maxflow-mincut algorithm, using as “source” the one or more entry points to $X$ and as “sink” the one or more exit points of $X$ (e.g., if $X$ is a merge tree, the “source” consists of several source segments at the leaves of the tree that correspond to arrivals and the “sink” corresponds to the arrival fix at the root of the tree).

**Pros:** The local MM is (in theory) easy to compute; we are able to modify existing software to do initial experiments. The metric accurately models the additional throughput capacity in the vicinity of the nominal route structure.

**Cons:** Bottlenecks in the constrained domain may result in a very low MM; thus, the metric does not account directly for “wide open regions” near the nominal route structure where potential holding patterns and other maneuvers may be possible. (An alternative “hybrid” metric that combines the MM with the others described earlier would address this weakness. Additionally, we can apply methods of [PPMK09] to explicitly model the capacity with the requirement for additional airspace for holding patterns, etc.)
B. Planning for Contingencies

We discuss a general mathematical model of our approach to planning for contingencies.

To motivate the discussion, we begin with a simple example. Figure 7 illustrates an example of a series of simplified space-time diagrams in the event of a non-cooperative aircraft. A set of trajectories representing (constant-velocity) SDO aircraft, evenly spaced along a flow, corresponds to a set of uniformly spaced parallel paths (line segments) in space-time (Figure 7(a)), with each trajectory satisfying speed constraints (which correspond to slope constraints in the space-time plot). In the example, we assume that the aircraft only use speed adjustments and holding patterns to avoid the non-cooperative aircraft without changing their originally planned routes. As shown in Figure 7(b), a non-cooperative aircraft crossing the flow was confirmed at time $t_2$ and is back under control at time $t_1$. The trapezoid represents the range in which the non-cooperative aircraft may show up at each time point between $t_2$ and $t_1$ (we use a trapezoid to model the fact that the uncertainty in position of the non-cooperative aircraft increases over time). The red trapezoid corresponds to a non-cooperative aircraft that is crossing the flow. Using speed control, with specified maximum and minimum speeds, the trajectories of the aircraft are adjusted in order to avoid the trapezoid. Some aircraft speed up and go “in front” of the trapezoid (corresponding to going “below” the trapezoid in space-time), while others slow down and go “behind” it (corresponding to going “above” the trapezoid in space-time). Figure 7(c) shows one way to avoid the non-cooperative aircraft: Aircraft 1 speeds up even before the non-cooperative aircraft was confirmed; aircraft 2 and 3 use holding patterns to wait for the non-cooperative aircraft, and aircraft 4 and 5 slow down slightly, thereby accommodating aircraft 2 and 3 as they reenter the flow after holding. (In Figure 7(c), aircraft 2 and 3 are shown with vertical segments in the trajectories, corresponding to waiting within a holding pattern; note that the scale of the space-time domain is not meant to be accurate of actual speed bounds.) The mitigation strategy for this specific scenario, involving the use of pure speed control and possible holding, maps to a 2D computational geometry problem in space-time: Determine a new set of trajectories around the off-nominal constraint (e.g., the red trapezoid), subject to speed bounds and available holding patterns.

![Figure 7](image-url)

Figure 7. View in space-time: Simple example of a flow of aircraft using speed control to avoid a non-cooperative aircraft crossing the flow.

More generally, the problem of planning for contingencies is that of planning for multiple flows of aircraft on a route structure (e.g., parallel flows or an SDO arrival tree) with the additional constraint: We plan for a specified set of possible events or off-nominal scenarios, each of which yields a temporary blockage (constraint) in the space-time domain. Within our mathematical model, all events are modeled in the same way: each event $E$ within an allowable set, $\mathcal{E}$, of possible events corresponds to a subset of space-time that represents an unexpected constraint or lack of resource availability within the space-time domain $D$. For example, a closed metering fix is an obstacle corresponding to the fix location, together with an interval of time during which it is unavailable; in space-time, the event is a right cylinder with the closed fix as the base (Figure 8(b)). As another example, a non-cooperative aircraft is an obstacle corresponding to the anticipated trajectory bounds of the non-cooperative aircraft; this gives a “tube” (or an “expanding tube”, to account for uncertainty in the trajectory) within space-time that must suddenly be avoided. An unforecast weather constraint also corresponds to a region within space-time that must be avoided.
Figure 8 shows an example of a series of space-time diagrams in the event of temporary blockage (e.g., a closed metering fix), modeled as a temporary (circular) constraint (Figure 8(b)) within the domain of interest $D$ (here, $D$ is simply a rectangle in the plane). Originally there were 5 aircraft planning to cross the sector. The event of the temporary closure causes the trajectories of aircraft 2, 3 and 4 to become infeasible, as shown in Figure 8(c), (d) and (e). Figure 8(f) shows a possible contingency plan for the 3 aircraft: Aircraft 2 uses an alternative route so that it goes around the cylinder; aircraft 3 flies as usual at first, and enters a holding airspace (represented by the vertical segment in its trajectory) to wait for the metering fix to be available again; aircraft 4 still uses its original route, but it slows down as soon as it enters the domain $D$ so that when it arrives at the metering fix location, the fix is already open again. We do not show in Figure 8(f) the trajectories of aircraft 1 and 5, which are not impacted.

Our mathematical model of planning trajectories within space-time follows the approach of [AMP10]. A “motion graph” approximates the free part of space-time. Each aircraft’s trajectory is a “tilted tube” through space-time whose center follows a path in the motion graph. Our goal is to plan a set of trajectories that is robust to the set $\mathcal{E}$ of possible events: An “adversary” can select events from the set $\mathcal{E}$ to serve as new, unexpected obstacles in space-time, and our plan must allow for the aircraft currently in the domain to transition to an alternate contingency plan. This may mean that our robust plan purposely has additional operational flexibility or that the spacing of aircraft along a flow is greater than it would need to be if not for the possibility of an off-nominal event. Naturally, we must make some assumptions about the set $\mathcal{E}$ of allowed events; we do not allow the adversary to suddenly surround and trap an aircraft with new constraints. The algorithmic problem, then, is to plan multiple tubes in space-time such that the trajectories are robust to the anticipated set $\mathcal{E}$ of events.

As in Section III, we classify the types of off-nominal events into two broad categories: (a) events caused by “special” aircraft (e.g. aircraft in emergency, non-cooperative aircraft); and, (b) events caused by airspace or airport resource unavailability. In most cases of type (a) off-nominal events, rerouting using global operational flexibility airspace is unnecessary. The aircraft in emergency have the highest priority and they usually use designated routes. Similarly, the aircraft that lost communication or non-cooperative aircraft have their own routes and their specific position at a time point can be pre-computed based on their original flight route and speed. SDO operations provide instructions for all affected aircraft so that they can safely avoid the “special” aircraft, e.g., by directing aircraft into

Figure 8. Planning for contingencies: A general mathematical model (in the event of a closed metering fix).
holding airspace or advising aircraft to make speed adjustments. In off-nominal events caused by “special” aircraft, it is often impossible to accommodate the “special” aircraft by simply letting the other impacted aircraft wait or adjust their speeds. In such scenarios or in situations where some specific airspace or airport resource is unavailable, e.g., a closed metering fix due to a large weather constraint, the SDO operations may have to redesign a substantial portion of the related route structure. The affected aircraft are directed to use topologically different alternative routes, which are usually within the global operational flexibility airspace of their original routes.

The natural algorithmic problem related to these events asks for the most efficient and safest way for all the affected aircraft. By efficient, we mean that the system performance is least affected and the SDO operations should minimize the number of impacted aircraft. Safety requires that all of the RNP requirements and miles-in-trail (MIT) requirements must be satisfied and that all weather constraints must be avoided. The problem requires solution algorithms that are employed in three phases:

1. First, determine if there exists a safe and efficient way for all of the affected aircraft to avoid the “special aircraft” by using only holding patterns and speed adjustments, without rerouting. This scheduling problem can be viewed as a slope-constrained multi-path routing problem in a 2D space-time domain, with one dimension being spatial (indicating the position of an aircraft in the routing structure – a one-dimensional manifold) and the other dimension being time. The output of the scheduling algorithm provides precise instructions for each pilot, e.g., how to adjust speed, whether to enter into holding or not, and provide pilots information about the holding pattern, e.g., the area, position and capacity. Further, the algorithm should guarantee that after the impact of the “special” aircraft, the system transitions to nominal condition as soon as possible.

2. Second, if no such speed-based solution exists (as determined by the algorithm), a local rerouting solution is sought, allowing reroutings within the local operational flexibility airspace. In addressing the rerouting problems that arise when scheduling via speed control and holding patterns is not an option, it is useful to consider the notion of contingency planning and Contingency Path Maps (CPMs): A CPM, is associated with each “type” i of contingency/off-nominal event (e.g. closed fix, closed airport, non-cooperative aircraft, unavailable airspace, etc.). At each point p in the CPM, the map gives the control law for how an aircraft currently at point p should respond to the off-nominal event. This captures the notion that the response to the event depends on the location of each aircraft in the nominal plan. The CPM may be constrained to use local operational flexibility airspace or global operational flexibility airspace.

A natural algorithmic problem associated with a CPM asks for the best alternative routes for the affected aircraft. The new routes must avoid weather constraints, stay within local/global operational flexibility airspace, and meet RNP requirements of different types of aircraft. Rerouting algorithms should provide efficient solutions, using the nearest possible alternative route and minimizing the number of affected aircraft. The alternative route is preferred to be within the local operational flexibility airspace of the nominal route, if possible; if not possible, then the route should be within the global operational flexibility airspace. (If at all possible, it is also preferred to stay within the same sector; this is captured already in the model of local/global operational flexibility airspace.) Depending on the types of responses to uncertain weather forecasts, the rerouting algorithm may take into account the uncertainty in its optimization. The deterministic approach is to have SDO operations provide pilots the alternative route information only when the existence of a possibly blocking weather constraint is confirmed. For an impacted aircraft, the algorithm computes the shortest alternative route that starts at its current location, goes around the blocking weather constraints, and rejoins the original route. Figure 9(b) illustrates a case in which a large weather constraint (shown in red, outlined in black) may possibly block the planned route. As soon as the constraint is confirmed, the approaching aircraft are directed to use an optimized alternative route that goes around the constraint, as shown in Figure 9(c).
An aircraft in emergency on route 3 defines a scheduling problem that is resolved by adjusting speeds of aircraft on routes 1 and 2.

A large potential weather constraint blocks the nominal route.

An alternative route is computed as soon as the latest weather forecast confirms the existence of the constraint.

The optimal overall routing scheme computed as soon as the $\theta$-probable constraint shows up in the latest weather forecast.

Figure 9. Algorithmic Approaches to Mitigate Off-Nominal Conditions.

A stochastic model of uncertain weather potentially allows SDO operations to provide pilots an alternative routing strategy that can be invoked as soon as a possibly blocking weather constraint shows up in the latest weather forecast. For example, if a “$\theta$-probable” constraint in the forecast is modeled as having probability $\theta (0 < \theta < 1)$ of actually occurring, then the algorithm for an impacted aircraft can compute the best rerouting strategy, optimizing, e.g., the expected total length of the rerouting solution, minimizing $\theta L_1 + (1-\theta) L_2$, where $L_1$ is the length of the alternative route in cases where the constraint materializes and $L_2$ is the length in the case that the constraint does not materialize. Figure 9(d) illustrates an example: As soon as the $\theta$-probable constraint is detected, the algorithm computes a routing scheme for all the approaching aircraft after point $A$, which will now fly along direction $AB$ instead. If later weather forecasts confirm the existence of the constraint, the aircraft will use route $ABDE$ as the alternative; otherwise, all of the impacted aircraft use route $ABCE$ to return to the originally planned route.

The contingency plans for a controller are based on an assumed off-nominal, emergency, or failure condition. For instance, if a metering fix becomes unavailable for an unspecified period of time, then the controller must have a contingency plan ready in advance. One possible CPM is based on the use of an Optimal Path Map (OPM), that can relocate all aircraft within a flow or along a branch of an SDO tree-based route planning solution to either an alternate airport or to another metering fix location. Figure 10 shows an example in which when emergency happens, aircraft 2 and 3 can be safely redirected using the given OPM to an alternate destination. But the OPM does not provide a prepared solution for aircraft 1, which thus needs a solution derived for its specific condition to avoid the emergency; e.g., aircraft 1 turns around and goes directly to the alternate destination.

The contingency plans for a pilot are based on the pilot’s role in the assumed off-nominal, emergency, or failure condition and the controller’s contingency plan solution. Each pilot is assigned a different role in the contingency plan solution based on the current state (location, altitude, RNP, weather constraints, etc.) of the system. For instance, if an aircraft declares a state of emergency, the aircraft would be taken off of the nominal SDO routing solution, with a feasible weather avoidance route directly to a runway (Figure 11), alternate airport, or metering fix to expedite landing of the aircraft. In such a case, the approach for setting up an emergency contingency plan is to have an automated FMS solution prepared 1 minute in advance, and have the FMS re-plan every minute, submitting
the latest contingency plan to the Air Traffic Service provider (ATSP) as the flight progresses forward. None of this preparation and submission of the contingency plan information needs to be performed “by hand” by the pilot and verbally communicated to the controller; rather, in NextGen it would be an automated process from FMS-to-ATSP computers.

Figure 10. The use of optimal path map for identifying routes to an alternate destination.

Figure 11. Use of an optimal route contingency plan for an emergency condition (outside the containment zone of the SDO plan), bypassing the metering fix, directly to the airport runway threshold.

V. Conclusions

In this paper, we discuss strategies for addressing off-nominal conditions in Super Dense Operations (SDO). Our main contribution is a mathematical and algorithmic formalization of the problem of planning for the unexpected, establishing contingency plans ahead of time for various types of off-nominal events. These formalizations will lead to new computational decision aid tools that will assist controllers and decision-makers in evaluating options to be ready for off-nominal conditions when and if they occur.

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