Maximum Flow Rates for Capacity Estimation in Level Flight with Convective Weather Constraints

Jimmy Krozel  
Metron Aviation, 131 Elden Street, Suite 200, Herndon, VA 20170
Joseph S. B. Mitchell  
Stony Brook University, Stony Brook, NY 11794
Valentin Polishchuk  
Helsinki Institute for Information Technology, P.O. Box 68, FI 00014
Joseph Prete  
Metron Aviation, 131 Elden Street, Suite 200, Herndon, VA 20170

ABSTRACT

We study the problem of computing maximum flow rates for capacity estimation of an airspace at a constant flight level in the presence of convective weather constraints, under various operational conditions. Our problem statements are for future Air Traffic Management (ATM) operations where jetway routing is removed and aircraft routes may conform with the geometry of weather constraints. We consider several ATM flow organizations: All Altitudes (aircraft flying in any direction), Alternating Altitude Rule (aircraft flying with headings from 0° to 180°, versus 180° to 360°), Monotonic Rule (aircraft flying in from the east), and Unidirectional (aircraft flying from east to west). We investigate both decentralized Free Flight scenarios and centralized Packed scenarios under each flow organization. For each flow organization, we compute the maximum flow rates for the airspace through experimental analysis based on simulated demand for travel through the airspace and routes computed using an algorithmic solution. We compare the throughput, observed in simulations, to the theoretical upper bounds we compute using network flow theory adapted to the geometric problem domain. We also compute a complexity metric to evaluate the solutions computed under each flow organization. Experiments are based on both real and synthesized weather data.

INTRODUCTION

Estimating the capacity of an airspace given a weather forecast indicating convective weather constraints is a fundamental problem in Air Traffic Management (ATM). If the demand for an airspace exceeds its capacity, then a traffic flow management control strategy for that airspace is necessary. The demand for an airspace is determined by the number and type of aircraft that desire to fly through the airspace within a particular time window. The capacity of an airspace is defined as the maximum number of aircraft per unit time that can be safely accommodated by the airspace, given controller and pilot workload constraints and airspace constraints (e.g., Special Use Airspace, convective weather constraints, etc.).

Capacity is determined by two main limiting factors: throughput rates determined by the geometry of the airspace and the airspace constraints, and human factors issues defined by pilot and controller workloads arising from sector counts and traffic complexity. This paper concentrates on limitations to throughput rates imposed by the inherent geometry of the airspace with weather constraints. By computing the maximum possible flow rates, we obtain a quantification of the “permeability” of a constrained airspace, separate from human factors issues affecting capacity. Since future ATM

1 This work was conducted while V. Polishchuk and J. Prete were Ph.D. students at Stony Brook University.
technologies may alleviate some of the bounds on capacity arising from human factors, it is important in considering capacity issues to study maximum throughput rates that are determined by constraints largely outside our control.

The primary focus of this paper is to investigate the tradeoffs between the airspace capacity and ATM flow organization in the presence of weather constraints. We compare a set of flow organizations that range from highly flexible to highly regulated. We examine both decentralized techniques, such as Free Flight [RTCA, 1995], and centralized control techniques, such as Flow-Based Route Planning (FBRP) [Prete and Mitchell, 2004; Krozel et al., 2007]. Furthermore, we examine the effects of platooning of aircraft within flows, ranging from platoon sizes of 1 (individual aircraft) through very large platoon sizes that approach continuous flows of aircraft following the same waypoints across the airspace. Platooning has been shown to result implicitly from decentralized control laws that pass two flows of aircraft through a common intersection location [Mao et al., 2001], and is also used to maximize capacity in the design of intelligent, hierarchical transportation systems for cars [Varaiya, 1993]. At constant flight level, a platoon is a set of aircraft, all of which are flying the same route (waypoints), while separated by a longitudinal Miles-In-Trail (MIT) requirement. For the purpose of establishing the maximum capacity, we set the MIT requirement to be the en route separation requirement of 5 nmi; however, if our algorithms are to be used for current ATM applications, a MIT requirement of 7, 10, 15, or 20 nmi may be appropriate. For the sake of simplicity, we have assumed that all flights are on a constant flight level, at one constant speed, and have the same Required Navigational Performance (RNP). The RNP establishes a constraint that all aircraft are equipped to fly within a fixed range from a route, thus insuring that air lanes can be placed next to one another without aircraft wandering into nearby air lanes.

This research has been conducted to support the Next Generation Air Transportation System (NGATS) [Swenson et al., 2006], which is also commonly referred to as NextGen. Tradeoff studies in this paper compare fundamental ATM flow organizations that are not dependent on today’s jet routes or current ATM practices. The study will help NGATS policy decision makers to choose between candidate ATM designs, and helps researchers understand the relationships between flow organization, capacity, and traffic complexity.

In today’s National Airspace System (NAS), there is a need to establish the estimation of capacity of an en route airspace to support Airspace Flow Programs (AFPs) [Krozel et al., 2006; Brennan, 2007] and to establish Flow Constrained Areas (FCAs). While the techniques presented in this paper do not depend on jet route structures that affect today’s AFPs and FCAs, our results do represent upper bounds that are useful for understanding the effect of hazardous weather constraints on AFPs and FCAs in the NAS. Today, there is no standard theoretical approach for establishing the capacity of a particular en route airspace in the NAS, particularly when impacted by severe weather; existing approaches are empirical. To this end, the theory and algorithms presented in this paper break new ground.

Related Work

Early work by [Schmidt, 1975] investigated the traffic variables, routes, sector geometry, and control procedures that contribute to a control difficulty index that empirically quantifies the workload required on the part of the air traffic control team to manage a sector. The sector capacity was limited by total workload as measured by the control difficulty index. The work of [Meyn, 2002] presented probabilistic methods for air traffic demand forecasting, including demand count probabilities for sectors. [Wanke et al., 2005] investigated probabilistic congestion management, including the prediction of traffic levels and airspace capacity. Their work warned that Monitor Alert Parameter (MAP) values should not be considered a measure of airspace capacity. Each sector is assigned a constant MAP, independent of the level of weather present in the sector, which identifies the peak number of aircraft that can be safely handled for a given 15-min time interval. The statistics of sector peak count prediction uncertainty was also studied in previous work of [Wanke et al., 2003]. The work of [Song et al., 2006] studied the
problem of predicting sector capacity for sectors in today’s NAS through a pattern recognition technique – recognizing the traffic flow pattern was included in their technique.

Generally, these previous methods of studying sector capacity stem from the empirical analysis of how controllers work in the NAS today, including the use of jet routes and controller workload limitations. In contrast to this, we describe in this paper a theoretical analysis of the airspace capacity as a function of hazardous weather constraints, independent of workload considerations and independent of today’s jet routes. The analysis is based on maximum flow rate concepts in geometric domains, developed by [Mitchell, 1990], [Mitchell and Polishchuk, 2007], and [Strang, 1983]. Our work is in support of the design of new roles for controllers and pilots in the NGATS, and addresses the maximum throughput of an airspace, assuming that workload is not a constraint.

Organization of Paper

First, we describe the modeling of the problem. Then, we discuss the theory of maximum throughput estimation. Next, we present algorithmic solutions, and compare them with the theoretical maximum throughput. Finally, we present our conclusions.

MODELING

Model of the Airspace

We consider a region of airspace specified by a constant flight level and a two-dimensional (2D) polygonal domain \( P \). Our experiments use a square airspace, however, all of our techniques apply immediately to general polygonal en route airspaces (e.g., sectors, centers, or FCAs). For synthetic-weather experiments the size of the square that represents the airspace is 60-by-60 nmi; for real weather experiments the size is 200-by-200 nmi, in order for the airspace to be large enough to exhibit intricate patterns of weather. One portion of the airspace boundary serves as the source while another portion serves as the sink. We restrict all flights to enter at the source and exit the airspace at the sink. We do not allow flow to originate or to terminate within the airspace (i.e., no aircraft exit or enter the flight level within the interior of \( P \)).

We make a number of simplifying assumptions. While these assumptions are not realistic for modeling today’s operations, they are made to establish a model with only a few parameters with a sufficiently close approximation to the most salient aspects of maximum throughput. First, we focus on a single flight level; aircraft are assumed not to change altitude within the airspace of the experiment, and cannot ascend or descend out of the airspace while within the experimental boundaries. Aircraft must enter and exit into the airspace on its (two-dimensional) boundary. All aircraft are assumed to have a constant speed of 420 kn in our experiments. (We note, however, that the FBRP software utilized in our experiments does allow any speed of aircraft and does allow speeds to vary by aircraft.) We also assume that all aircraft meet an RNP requirement of 5 nmi, so that they are laterally separated by 5 nmi, and we assume also a 5 nmi MIT requirement. As a result of these MIT and speed requirements, at peak throughput, a single lane of traffic can carry 84 aircraft/h past a particular point in space. We assume that aircraft can come arbitrarily close to hazardous weather as long as they do not enter it; our algorithms readily permit safety margins to be added to the hazardous weather regions, but our experiments used a margin of zero. We ignore the earth’s curvature, since it is not a significant factor over relatively small experimental areas.

Convective Weather Constraints
We consider constraints that arise from convective weather. Convective weather severe enough to pose a safety hazard for aircraft is often characterized by specifying an intensity threshold in the National Weather Service (NWS) scale. While the criteria for weather avoidance depend on pilot preferences and airline guidelines, research [Rhoda and Pawlak, 1999] shows that pilots generally avoid NWS Level 3 and higher weather cells (greater than 13.3 mm/hr rainfall or reflectivity greater than 41 dBZ). While the altitude of cloud tops in severe storms is also an important factor [DeLaura and Evans, 2006] that pilots consider in determining which storm cells to avoid, cloud tops data are not included in our algorithmic experiments. (For today’s fixed jet route structure, [Martin et al., 2006] show that ignoring storm echo tops in determining blockage of jet routes could result in overestimating the number of blocked routes by a factor of two.)

The Weather Severity Index (WSI) for en route airspace is defined as the percentage of the airspace that is occupied by convective weather with NWS level 3 or greater. We acknowledge that the structure of weather cells, not just their number, can have a significant impact on the throughput [Mitchell et al., 2006]. When generating synthetic weather datasets, we generate weather cells according to a common distribution, across all severity levels, so that the structure of cells can be expected to be similar even as severity is varied. All weather used in our experiments is static. (Note, however, that the FBRP routing methods we employ do apply to the dynamic weather constraint environment, as shown in [Prete and Mitchell, 2004].)

**Synthetic Weather Generation**

Our experiments use synthetic weather, randomly generated in order to have a wide variety of coverage (WSI). To generate random "popcorn" weather, we employ a simple model in which random circular weather cells are generated within a larger airspace (specifically, within an 80-by-80 nmi square centered on the 60-by-60 nmi airspace of interest). In our simple model, each formation's center is uniformly (independently) distributed in the region; the sizes vary between 2 to 6 nmi in radius, according to a symmetric triangle distribution with a peak at 4 nmi. Weather cells are randomly added or removed until the measured WSI is within a small percentage of the desired value. For each WSI value from 0% to 70% in 5% increments, ten weather samples are generated. No experiments were run for WSI over 70% because throughput approaches zero at WSI values greater than 70%.

**Real Weather**

Our experiments also include real-world weather data. We use samples that are roughly 200-by-200 minutes (of latitude/longitude). (In order to approximate more closely a square region, we slightly stretch the samples east-west to cover a full 200 nmi). The data samples used in the experiments come from NWS data for certain time slices on severe weather days: June 26, 2002 and June 27, 2002. These time slices have been subjectively selected as the times of greatest spatial coverage for those particular days. In order to extract multiple samples of weather data, we shift a 200-by-200-minute (i.e., 3.33 degree) sampling region across the US by 100-minute steps. The WSI is measured for each sample, allowing us to create a table of many instances of weather at various WSI values.

Real weather at 200-nmi scale does not normally have very high WSI, even though it may represent a significant navigational constraint. In order to obtain a wide variety of weather samples with high WSI values, we examine the weather data twice, once using a NWS level 3 threshold, and once using a NWS level 2 threshold. In this way, we are able to extend our results to realistic weather samples with WSI values as high as 34%.

**ATM Flow Organization**

We examine four different ATM flow organizations in order to determine how maximum throughput and complexity are affected by the structure of the flow. The rules are illustrated in Figure 1. Each rule represents a different tradeoff in throughput, freedom of routing, and human factors complexity for air traffic controllers. In order to avoid cases when a flight “clips a corner” of our square airspace, in our
simulations we require flights to enter and exit the airspace within the central 75% of each side of the airspace; this requirement is indicated with thickened boundary lines in Figure 1. In order, from least restricted to most restricted, the ATM flow organizations are:

- **All Altitudes (AA)**. Flights cross the airspace in any direction, entering on any side and exiting on any side as long as the entry and exit sides are different.

- **Alternating Altitude Rule (AAR)**. Flights follow the standard rule [Spence, 2007] that east-to-west aircraft are altitude separated from west-to-east aircraft according to “east is odd, west is even”. Our experiments using the AAR permit flights to cross in any direction, as long as the exit point for the flight is east of the entry point. The entry and exit sides must be different, and no flight may enter on the east side or exit on the west side.

- **Monotonic Rule (MR)**. Flights must enter the airspace on one side (the west) and may exit at any point on the other three sides; thus, as with AAR, flights are required to be monotonically eastward, but, in the MR case, they must enter the airspace on one specific side (the west side).

- **Unidirectional Rule (UR)**. Flights must enter the airspace on the west side and exit the airspace on the east side.
Figure 1. Four ATM flow organizations.

Decentralized Free Flight versus Centralized Packed Demand

In our experiments we investigate demand sets for both decentralized Free Flight scenarios and centralized Packed scenarios. In Free Flight scenarios, flights may enter and exit the airspace at arbitrary points on the boundary of the airspace, as long as the ATM flow organization of the experiment is obeyed. In Packed scenarios, flights must enter and exit the airspace at predetermined points, specially spaced along the boundary of the airspace, based on a calculated route packing designed to maximize throughput. By properly spacing the entry/exit points, one can avoid having many wasteful gaps between air lanes.
Metrics of Comparison

Potential airspace utilization and the traffic produced by our algorithms are analyzed with two quantitative measures: throughput and airspace complexity.

**Throughput**

Throughput is measured in terms of the number of aircraft that enter the airspace $P$ within the time window of the experiment. Any aircraft that is permitted to enter is required to have a specific path that avoids hazardous weather. In order to measure throughput, an aircraft is considered to have a point position, so it has either entered the airspace completely in the experiment time window or not at all. Experiments use a 30-min time window.

Both when routing aircraft and measuring airspace throughput, we use an RNP value of 5 nmi (meaning a 5 nmi horizontal radius from the route centerline), and air lanes impose a required MIT of 5 nmi as well. Since we are measuring airspace throughput based solely on hazardous weather constraints and the ATM flow organization in effect, it is assumed that any routed air lane is completely full; under our assumptions, this corresponds to 84 aircraft/h. In current practice, aircraft are never packed so tightly for reasons of safety and maneuverability.

**Airspace Complexity**

The complexity metric we utilize was originally defined in [Krozel et al., 2007]. It is defined at points of a regular square grid within an airspace. The complexity at grid point $p$ is defined in terms of a weighted average of the variance of the velocity vectors of aircraft in the neighborhood of $p$, scaled according to the distance of the aircraft from $p$. Specifically, for some radius $R$, for each aircraft $a_i$, a scaling factor, $s(p)$, is introduced:

$$s(p) = \frac{\max(R - d(a_i, p), 0)}{R}$$

where $d(a_i, p)$ is the distance from $a_i$ to $p$. This scaling implies that the contribution of an aircraft to the airspace complexity at $p$ falls off linearly, from 1 to 0, with distance from $p$, up to the radius $R$. Flights for which $d(a_i, p)>R$ are not considered in the calculation of the complexity for point $p$. (Other scaling functions, such as a Gaussian, could be applied as well, as long as the function smoothly decreases to zero, or close to zero, at radius $R$.) The average contribution of all aircraft is computed, with aircraft $a_i$’s contribution scaled according to $s(p)$. For an instant in time $t$, we define the average local velocity and variance of velocity as follows. Let $V_{avg}(p,t)$ be the local average velocity vector in the neighborhood of grid point $p$ at time $t$, scaled according to the factors $s(p)$:

$$V_{avg}(p,t) = \sum s(p) v_i(t)$$

where $v_i(t)$ is the velocity (vector) of aircraft $a_i$ at time $t$. The (scalar) quantity $\left\| v_i(t) - V_{avg}(p,t) \right\|^2$ gives the squared deviation of the velocity of aircraft $a_i$ from the local average velocity vector in the neighborhood of grid point $p$ ($\|u\|$ is the Euclidean length of vector $u$). The larger this quantity, the more variation there is in the velocity vectors, as contributed by aircraft $a_i$ in the neighborhood of point $p$. Summing over all aircraft, and scaling by $s(p)$ to account for the distance from point $p$, we obtain the expression for the scaled-contribution velocity variance at point $p$, at time $t$:

$$Var(p,t) = \sum s(p) \left\| v_i(t) - V_{avg}(p,t) \right\|^2.$$  

Our complexity metric is based on a linear combination of this variance term and a density term, $N(p,t)$, defined to be the number of aircraft at time $t$ within distance $R$ of point $p$. The overall composite complexity metric takes into account velocity variation and density:
\[ C(p,t) = \lambda_1 \sum s_i(p) \| v_i(t) - V_{avg}(p,t) \| ^2 + \lambda_2 N(p,t). \]  

In our experiments, we use \( \lambda_1 = 0.36 \), \( \lambda_2 = 2 \), and \( R = 35 \) nmi. The overall complexity at time \( t \) in a given region of airspace is obtained by summing \( C(p,t) \), over all grid points within the region. (Alternatively, we may use the maximum value instead of the sum of values, so that a single hotspot is more significant than a large region of lesser complexity; this may be valuable in some applications, but here we chose to examine the aggregated complexity represented by the sum.) Our use of complexity in this paper requires a single number for comparison over all scenarios, so we measure the overall complexity in a region of airspace at one-minute intervals and then average across the intervals.

**THEORY OF MAXIMUM THROUGHPUT**

In this section, we investigate the theoretical maximum throughput of an airspace in which there are given deterministic weather constraints.

**Flows in Discrete Networks**

First, we review some basic definitions and facts about network flows. A network is a directed graph \( G=(N,A) \), where \( N \) is the set of nodes and \( A \) is the set of (directed) arcs connecting certain pairs of nodes; each arc \( e \) has a capacity, \( c(e) \). Two nodes, \( s \) and \( t \), in \( N \) are designated as the source and sink, respectively; all other nodes of \( N \) are internal nodes. A flow in \( G \) is an assignment of a flow value, \( f(e) \leq c(e) \), to each arc \( e \) in \( A \), such that the total flow into each internal node is equal to the total flow out of it. The value of the flow is the total flow out of \( s \); by flow conservation, this is also the total flow into \( t \).

The maxflow problem is to find a flow with maximum value. A cut in \( G \) is a partition of the nodes into two sets \( S \) and \( T \), such that \( s \) is in \( S \), and \( t \) is in \( T \). An arc \( e \) is said to cross the cut if one of its endpoints is in \( S \), and the other is in \( T \). The capacity of a cut is the sum of the capacities of the arcs that cross it; no flow in \( G \) can possibly have a larger value than the capacity of any cut. A mincut is a cut of minimum capacity, and therefore its capacity is an upper bound on the value of any flow. It is a basic fact in optimization that the capacity of a mincut equals the value of a maxflow; this “maxflow/mincut” theorem is a consequence of duality in linear programming [Ahuja et al., 1993]. Furthermore, efficient (polynomial-time) algorithms are known for computing maximum flows and minimum cuts.

**Continuous Flows with Deterministic Constraints**

Notation and Structure of Continuous Flows

The notions pertinent to discrete network flows can be extended naturally to flows in 2D domains. Instead of a discrete network, a continuous domain, such as a simple polygon \( P \), is considered (Figure 2). Two boundary edges, \( s \) and \( t \), of the polygon are designated as the source and the sink. A flow \( f \) in \( P \) is a vector field. The constraints \( H_1...H_K \) are pairwise-disjoint simple hazard polygons that lie fully inside \( P \); the flow is not allowed to pass through any of the constraints, i.e., for any point \( x \) within a constraint, \( f(x) = 0 \).
The theoretical maximum throughput of a continuous flow field is determined by the $s$-$t$ mincut. The polygon $P$ is assumed to be uniformly capacitated, i.e., the length of the flow vector must nowhere exceed 1. The value of the flow is defined as an integral of the normal component, $fn$, over $t$. There are no sources or sinks inside $P$; i.e., for any $x$ in $P$, $\text{div} f(x) = 0$.

The maxflow problem is to find an $s$-$t$ flow of maximum value. A cut in $P$ is a partitioning of the polygon into two parts so that $s$ is in one of the parts, and $t$ is in the other. The capacity of a cut is the length of the boundary between the parts, where only the part of the boundary that is interior to $P$ (and not on the boundary of $P$ or within a constraint) is included in the length. We use the term “mincut” to refer to the path(s) within $P$ that comprise the boundary of the cut (shown as the dashed paths in Figure 2), as well as to refer to the capacity (length) of the mincut.

The maxflow/mincut theorem holds for polygonal domains [Strang, 1983], as it does for discrete networks. [Mitchell, 1990] developed geometric shortest path techniques to compute the maxflow and the mincut efficiently in 2D polygonal domains, even if there are multiple source and sink edges on the boundary of the domain. In this paper, we are concerned with computing a mincut in a polygonal domain, since it represents the maximum theoretical throughput of an airspace with respect to a given set of weather constraints.

**An Algorithm to Compute a MinCut**

The source and sink edges, $s$ and $t$, split the boundary of $P$ into two polygonal chains, which we denote by $B$ and $T$ (Figure 3). If $s$ and $t$ represent the west and east boundaries, then $B$ and $T$ are the bottom and top (including the small corner exclusion boundary segments).

The critical graph of the airspace has a vertex for each constraint, for $B$, and for $T$. The critical graph joins a pair of vertices with an edge whose length is equal to the minimum (Euclidean) distance between the constraints corresponding to the vertices. An example is shown in Figure 3. Dashed edges in Figure 3 correspond to pairs of constraints for which the minimum distance is achieved by a line segment that passes through other constraints; such segments are not necessary in the critical graph, as they will never be part of a mincut (as seen by a simple application of the triangle inequality). The mincut corresponds to a shortest $B$-$T$ path in the critical graph. [Mitchell, 1990] and [Gewali et al., 1990] showed how to use computational geometry techniques to compute a mincut (and a corresponding maxflow) more efficiently than naively constructing the critical graph and searching it; however, those techniques require a more complex implementation, which we do not do here.
Flows with RNP Requirements

In our ATM model, the polygon $P$ represents the airspace, $s$ and $t$ represent the edges through which the aircraft may enter/exit $P$, and the constraints correspond to hazardous weather. The modeling of weather constraints may be in any form (polygons, grid cells, or circles).

Each air lane is thought of as a thick path, where the thickness of a path equals the RNP. The problem of computing the maximum number of air lanes from $s$ to $t$ through $P$ is equivalent to searching for the maximum number of thick paths that can be threaded though the airspace from $s$ to $t$. This problem is closely related to the continuous maxflow/mincut problem; however, there is an important distinction due to the discrete nature of routing an integral number of air lanes. As with the continuous maxflow/mincut computation, we can write the problem as a shortest path problem in the critical graph defined previously; however, before computing the shortest path in the critical graph, the length of each edge is rounded to $\lceil l_{ij}/\text{RNP} \rceil$, where $l_{ij}$ is the distance between constraints $i$ and $j$, and the lower brackets denote rounding down to the nearest integer. Rounding edge lengths to a multiple of RNP reflects the fact that only the integer number of air lanes, equal to the rounded length of an edge, may be routed through the edge while meeting the RNP requirement.

Maximum Throughput as a Function of the ATM Flow Organization

In the maxflow/mincut discussion above, the traffic is allowed to enter the airspace only through the west side, and exit only through the east. This corresponds to computing the mincut appropriate for the Unidirectional Rule. Refer to Figure 4 (b).

For the other ATM flow organizations, the mincut is defined slightly differently. For the Monotonic Rule, the bottom $B$ and the top $T$ used in defining the critical graph correspond to the L-shaped (artificial) constraints placed in the northwest and southwest corners of the airspace in order to restrict aircraft from entry/exit very near the corners. See Figure 4 (a). The corresponding mincut represents a theoretical upper bound on the number of air lanes that can be routed according to the Monotonic Rule, entering the airspace between the L-shaped corner boundary constraints.

For the All Altitudes and Alternating Altitudes Rules we must proceed differently in obtaining a theoretical upper bound, since flights may enter or leave the airspace through any of the four sides. First, we compute the mincuts associated with each of the four sides of the airspace, between each pair of consecutive corners. In Figure 4 (c), the corners are labeled $A$, $B$, $C$, and $D$, and mincuts are shown connecting pairs of consecutive corners; we let $x_{AB}$, $x_{BC}$, $x_{CD}$, and $x_{DA}$ denote the respective lengths of these cuts. (Note that two distinct mincuts may partially coincide.) Under our assumption that flights are not allowed to enter and exit through the same side, each flight must cross two of the mincuts. Thus, an upper
bound on the number of air lanes is given by $\frac{1}{2}(x_{AB} + x_{BC} + x_{CD} + x_{DA})$. This value is the theoretical maximum throughput for both the AA and the AAR rules.

Figure 4. The mincuts and flows under each of the ATM flow organizations.

(a) Monotonic Rule (MR)

(b) Unidirectional Rule (UR)

c) All Altitudes or Alternating Altitudes Rule (AA or AAR)
ALGORITHMIC SOLUTION APPROACHES

An important feature of the mincuts that we compute is that they give tight estimates of the maximum throughput under each ATM flow organization. This means that, from maxflow/mincut theory, we know that the mincut is equal to the number of air lanes that can actually be routed through the airspace. Our experiments compute the mincut values (exactly); from the theory, we know that we could implement an algorithm that achieves a routing of the number of air lanes, equal to the mincut value.

While mincut algorithms provide a hard theoretical upper bound on the throughput of a region of airspace, this bound is tight only if several assumptions hold: all aircraft are routed in non-crossing lanes, all constraints are static (not changing in time), and all aircraft are centrally controlled to avoid conflicts. In some cases, one wants the option to use crossing lanes of traffic in order to satisfy demand. For example, during the passing of a densely formed group (“platoon”) of north-south flights, east-west traffic may be blocked; however, after the group passes, there is an opportunity for east-west demand to be met. In high-density situations, flight paths can also block each other from making optimal use of the bottlenecks in an airspace. For this reason, we sought an experimental method for estimating the practical throughput of a region of airspace.

In our experiments, we use the FBRP routing algorithm of [Prete and Mitchell, 2004] to determine routes, recognizing that, due to its heuristic method of incrementally adding routes (described below), it is not guaranteed to achieve the theoretical maximum throughput. We use the FBRP because it is a more general routing tool than is known theoretically for routing maxflows. In particular, FBRP is capable of routing flows in the presence of moving weather constraints and of imposing certain turn and heading constraints on routes. It is also capable of routing small groups of aircraft as “flows”, or single aircraft (as in Free Flight). These features of FBRP make it more applicable to the ATM domain than the relatively limited theoretical algorithms ([Mitchell, 1990]) that achieve maximum throughput, since those algorithms apply only to static weather constraints and do not apply to flows with heading constraints.

**Estimating Throughput with the FBRP Algorithm**

The FBRP algorithm is an incremental algorithm that computes flows (routes that are available for a specified window of time) between specified start and end positions on the boundary of the airspace. It computes each route in succession, while avoiding hazardous weather and all previously-routed aircraft. Technical details of the algorithm are given in [Prete and Mitchell, 2004] and [Prete, 2007]. For our experiments, we add an elliptical constraint to avoid excessive rerouting. Specifically, we require each route to stay within an ellipse whose foci are the start and end points and whose defining distance (which is the sum of the distances from a point on the ellipse to each of the two foci) is equal to the Euclidean distance between the foci, plus a parameter $E$ ($E = 10$ nmi for synthetic weather scenarios and $E = 30$ nmi for real weather scenarios).

The demand across the airspace is determined by entry/exit points on the boundary of $P$. For both the Free Flight and the Packed scenarios, we first calculate the set of points along the boundary that are free of weather and other constraints, at 0.1 nmi intervals; we then eliminate points in order to achieve the desired spacing of points, usually a minimum of 5 nmi between points (but sometimes a larger spacing in order to maximally spread points).

For decentralized Free Flight scenarios, we experimentally measure airspace throughput by randomly generating very large amounts of demand until it is no longer possible to route more aircraft across the airspace. We start at the beginning of the time window for the scenario, iterating through possible timestamps until we reach the end. (The number and frequency of chosen timestamps are based on the platoon size.) For each timestamp, we generate random demand according to the ATM flow organization, trying to route it using the remaining free airspace, until 40 demand requests in a row have failed. All unblocked points in that time slice have an equal chance to be chosen as entry or exit points, depending on the ATM flow organization. For any given timestamp, the same entry point is never used twice.
For centralized Packed scenarios, we use the set of unblocked candidate entry/exit points somewhat differently. For the All Altitude and Alternating Altitude Rules, we pair off points starting from the corners (all four simultaneously) and working towards the center; all of these routes cut across a corner, and routes closer to the corners have priority over those further away. Once we can no longer pair off corner points, we pair off the remaining candidate entry/exit points directly across the sector. For the Unidirectional and Monotonic Rules, we calculate the set of points along the appropriate entrance and exit sides, such that we have an equal number of entry and exit points, the entry points are all equally spaced, and the exit points are all equally spaced (these can be different spacings, and usually are). The points are then paired off from the center outwards, giving the center routes priority over the border routes. While there is no guarantee that the produced routes will achieve the theoretical maximum, they come close, as will be seen in the results, and the FBRP is capable of maintaining certain operationally useful constraints on the generated routes, such as the number of turns, the monotonicity with respect to prescribed directions, etc. We examine the resulting routes computed and directly measure throughput and complexity of the airspace as described above.

**Platooning**

A *platoon* is a set of aircraft, all of which are flying the same waypoints, separated by the MIT requirement. This means that a single solution to the routing problem is reused by multiple aircraft. Platoon sizes in our experiments (e.g., as illustrated in Figure 5) range from one aircraft (no platoon) to 100 (which closely resembles a continuous flow). Between these two sizes, platooning represents a compromise between letting flights go anywhere they want and keeping them highly organized in a flow. With platooning, a small number of aircraft are grouped together, and since they can be treated as one unit they are simpler to route and to monitor. The disadvantage is that platooning aircraft must have the same arrival and destination point within any given airspace.

With platooning, demand is generated at time intervals of half the time it takes for a platoon to enter the airspace. For example, a platoon for 10 aircraft is 50 nmi long (given an MIT of 5 nmi and including the MIT after the last aircraft of the platoon), and therefore takes 7.14 min (50/420 h) to enter the airspace; thus, demand is generated at 3.07 min intervals. Our experiments do not include mixed platoon sizes; within a single experiment, all platoons were the same size.

![Figure 5. Different levels of platooning.](image)
EXPERIMENTAL RESULTS

Experiments with Synthetic Weather

Figures 6 and 7 show plots of throughput as a function of WSI. The plots include mincut values, which give the theoretical upper bound on the throughput if air lanes are perfectly packed, as given by the maximum flow.

The throughput data shows a number of trends. Throughput decreases with increasing weather severity index – a highly intuitive result that our data confirms. Throughput also decreases with increased levels of flow organization (see Figure 6 for an illustration of both of these trends). The All Altitude and Alternating Altitude Rules achieve the highest throughput in all tests, in part because the greater freedom of the flow organization scheme increases the size of the border on which flights can enter and leave. Monotonic and Unidirectional Rules, by contrast, show much less throughput in almost all cases. These results are reflected in both the experimental results and the theoretical values. The reduced throughput is caused by the entry constraint imposed by the rules: flights following the MR/UR can only enter from one side, while flights following the less constrained AA/AAR can enter and exit from virtually any pair of sides. Based on this property, we estimated that AA/AAR throughput should be about double that of MR/UR, and this seems to be true in the majority of the cases.

Throughput generally increases when switching from a disorganized Free Flight scheme with random demand to an organized Packed scheme. The Free Flight demand in the scenario is inflexible, reflecting that individual aircraft have specific routing requirements to fulfill. This means that flights and platoons tend to block each other once routed, resulting in an airspace that gradually fills up with somewhat randomly-oriented routes. The discrepancy disappears at higher WSIs (40% and above), reflecting that at such high weather coverage, there are few candidate air lanes to begin with, and the first aircraft to arrive that wants to travel in the available direction will take it. In general, however, only Packed scenarios come reasonably close to reaching the theoretical limit on throughput (Figure 7b).

Platooning has positive effects on throughput in all cases. Larger platoon sizes lead to greater throughput; this trend is much stronger with systematic packing, but it is also present in the Free Flight cases. The largest positive effects are seen in moving from no platooning – a platoon size of one – to platoons of two or three aircraft. Increasing platooning from three to higher numbers of aircraft (10-100) does not result universally significant throughput gains. However, significant gains are found in large platoon sizes in two cases: when combining the two most flexible flow organization schemes with systematic packing. In these cases, platooning continued to produce gains up to 20-50 aircraft per platoon.

The mincut dependence on the WSI is different for each flow organization. As mentioned above, the weather constraints in our experiments were generated at random uniformly over the square, thus modeling popcorn convection. It is known that in the presence of popcorn convection, the mincut drops roughly according to $1 - \sqrt{WSI}$ [Mitchell et al., 2006]. This is evident in the plots for the unidirectional flows. For the other rules, the drop was closer to linear. This is due to the fact that in these rules the mincut line was always going close to the “corners” of the square (the bottom B and top T, as in Figure 4). Thus, although the weather cells were generated over the whole square, only a relatively thin region close to the corners impacted the mincut value. This is a feature of a squall line weather organization. As confirmed in our earlier work [Mitchell et al., 2006], squall lines lead to a linear decrease of the mincut with WSI.

There is a small non-monotonicity of the MR-mincut, due to the random weather generation, but this is just a statistical fluctuation in randomly generated inputs.

Figure 8 shows the complexity of the airspace in these experiments. An increase in flow organization (AA to AAR to MR to UR) produces a decrease in complexity, as would be expected. An increase in platoon size also produces a decrease in complexity. The larger platoon size enforces more
order on the flights overall, since all flights in a platoon follow the same path, moving at the same
velocity. The only case when the complexity increased when going to higher platoon sizes is when the
platoon size goes from 1 up to 2 or 3. The reason is that the complexity has two parts: the proximity of
aircraft and the variability of aircraft velocity vectors. When there is no platooning (platoon size is 1), the
first term is negligible in comparison to the same term in the presence of platooning. The change in
importance of the proximity term explains the increase in complexity when going from platoon size 1 to
platoon size 2 to 3, but the complexity values between these platoon sizes are in general comparable.

When WSI increases beyond 30%, throughput decreases, causing decreases in complexity. Additionally, an increase in WSI reduces the number of available routes and tends to separate flights more, resulting in less variance of direction in the neighborhood of any given flight; at high WSI values, the majority of flights in the neighborhood of any point are likely to be from one platoon travelling in one
direction. However, below 30% WSI, complexity tends to decrease instead of increasing. Below that
value, the weather becomes a less significant obstacle and flights are likely to take straighter paths to their
destinations, decreasing complexity.

![Figure 6. Throughput (aircraft/h) and Complexity (unitless) vs WSI (unitless) comparison of all
methods for synthesized weather.](image)

(a) Decentralized Free Flight Demand  
(b) Centralized Packed Demand
Figure 7. Throughput (aircraft/h) vs WSI (unitless) for synthesized weather.

(a) Decentralized Free Flight Demand  (b) Centralized Packed Demand
Figure 8. Complexity (unitless) vs WSI (unitless) for synthesized weather.
Note that platoon size has little to no effect on complexity in Packed scenarios, especially with more controlled flow organizations and with higher WSI s. Platoons tend to follow the same routes in Packed scenarios because there is little reason to deviate from the existing shortest path, so in practice small platoons tend to produce as little complexity as large ones under these conditions.

We examine the dependence of the complexity of the solutions on the ATM flow organization in order to determine whether the choice of a particular flow organization would have a significant impact on flight controller workload. We find that, as a general rule, complexity increases substantially when the directions of flights are permitted a larger degree of freedom. When flights can cross in any direction, the resultant situation is theoretically harder for a controller to monitor than when only east-to-west flights are permitted. Complexity generally decreases with larger platoon sizes because larger platoons enforce more structure on aircraft routing.

Experiments with Real Weather

The real-weather experiments use roughly 200-by-200 nmi regions of airspace and a platoon size of 40. The maximum theoretical throughput of a 200-nmi-square region of the airspace, under the Unidirectional or Monotonic Rules, is 2604 aircraft or 31 lanes, assuming that all aircraft are traveling directly west-to-east with no weather constraints and maximally packed. Under the All-Altitude or Alternating Altitude Rules, the maximum is 5208 aircraft or 62 lanes, under equivalent assumptions of perfect packing into the airspace.

When routing aircraft in parallel flows using the Unidirectional or Monotonic Rules it is found (Figure 9) that even small fractions of weather (WSI = 10%) can cause disturbances in the ability to route the maximum possible number of aircraft. Typically 40-60% of the clear-weather throughput of an airspace is actually reached in such cases.

This is not the case, however, for the All Altitude or Alternating Altitude Rules. Both the mincuts and the experimental data suggest a more gradual falloff in throughput as the airspace increases in the amount of weather severity. Since real weather at the 200-nmi scale tends to form into large walls of hazardous weather, these walls tend to significantly block either the east or west borders, making unidirectional flight impossible, but if flights are allowed in all directions, two or three out of the four border lines will typically be mostly unblocked, and flights can still be routed.

The characteristics of the synthetic “popcorn” weather differ markedly from that of real weather. In particular, the real weather we use in our experiments is non-uniformly distributed across the airspace. The typical result (Figure 10) of this in the real-weather experiments is that there are only a few gaps in the airspace hazardous weather constraints, allowing through only a few lanes of traffic. However, the real weather in our experiments is not considered a squall line. When the area is relatively clear, there is little difficulty in routing many aircraft. This is partially the result of the greedy nature of FBRP routing; one platoon is routed at a time, along the shortest possible path, and this tends to block passages between neighboring hazardous weather constraints. As above, this tends not to affect All Altitude and Alternating Altitude Rules very much, because there are still many options for aircraft to enter and exit the airspace.

Real weather experiments show the same complexity trends as do the synthetic weather experiments, for the same reasons.
Figure 9. Throughput (aircraft/h) and complexity (unitless) versus weather severity (unitless) for real-weather samples.
CONCLUSIONS

This paper presents both a theory and a practical method for airspace maximum throughput estimation. The focus is on the inherent throughput limitations implied by the geometry of airspace constraints, taking into account separation of aircraft from each other (by establishing air lane widths for flows) and separation of aircraft from weather (by avoiding passing through or too close to hazardous weather) or other airspace constraints. The inherent throughput limitations are one component of capacity estimation:

Figure 10. Typical solution for real weather data over 200-by-200 nmi of airspace.
The capacity of the airspace cannot exceed the throughput rate upper bound given by the geometry of the constraints. The other important component of capacity estimation is controller workload: Upper bounds on sector counts and the complexity of air traffic place limits on the capacity, constraining the number of aircraft that can be routed through a sector. Our results can be used in conjunction with human factors considerations impacting controller workload to obtain capacity estimations in the presence of hazardous weather.

Our study investigates the tradeoffs between: (1) throughput estimations under ATM flow organizations that include both decentralized Free Flight and centralized Packed methods, and range from highly-flexible to highly-organized control – spanning aircraft flight in all directions, to aircraft following an alternating altitude rule, to aircraft flying only in one direction (e.g., west-to-east), (2) throughput estimation as a function of weather severity ranging from no weather constraints to severe weather constraints that make the airspace impassible, (3) flows that range from platoon sizes of one (individual aircraft) to very large platoon sizes that approximate continuous flows, and (4) the complexity of the resulting traffic flows. Our study makes a number of simplifying assumptions in order to reduce the complexity of analysis without significantly altering our conclusions: all flights are traveling at the same speed, under the same RNP (lane width), and at one flight level, without considering altitude changes or pop-ups from neighboring altitudes.

We find that the theoretical maximum throughput of an airspace is quite high compared with the estimated throughput of Free Flight methods of crossing the airspace, and that organization of flows utilizes the airspace much more thoroughly, at a lower complexity, but at the cost of enforcing heavy constraints on all flights within the airspace. In general, the use of platooning increases the throughput for each ATM flow organization, however, with diminishing returns after platoon sizes of two or three aircraft in a platoon. This is not true for low-weather coverage cases of All-Altitude and Alternating Altitude Rules, however; platooning continues to provide moderate benefits when increased past a size of three. Maximum throughput is most accurately measured via theoretical mincut methods, when applicable, but our theoretical bounds are tight only for static weather and strict flows from one side of an airspace to the other. Experimental methods using algorithms to maximize throughput (as with centralized Packed scenarios) are capable of closely approximating the mincut measure. Our experiments demonstrate that some planning component is a requirement for efficient utilization of airspace, but that this becomes less important as the severe weather coverage rises and the amount of free space available reduces the improvement that planned routing can achieve.

Future Work

The theory for maximum-throughput flows described here was developed, and the experiments run, under various assumptions for simplicity in the initial study. Future work will address several important ways in which many of these assumptions may be lifted.

1. **Different RNP and MIT requirements.** Future work will remove the assumption that all aircraft arriving at the entry to the airspace have the same RNP and MIT requirements. In a real-world scenario, aircraft with different navigation capabilities (and therefore able to comply with different RNPs) are likely to be routed through an airspace. While the FBRP algorithm [Prete and Mitchell, 2004] does apply more generally to flows with different air lane widths (for different RNPs), the mincut theory developed here does not apply immediately to flows with different air lane widths. Future work will address the computational complexity of computing maximum possible throughput and corresponding routes for flows of aircraft with different air lane widths. We expect that, while the general problem is likely to be computationally intractable, special cases (e.g., having only a few different widths) may be efficiently solvable with a dynamic programming variant of our algorithm.

2. **Different speeds.** Future work will generalize our throughput bounds to handle aircraft that fly in different speed categories. While maximum throughput is indeed bounded by that computed with our constant-speed assumption (assuming maximum possible speed of any aircraft), more
realistic bounds and estimates will be based on throughput for a given distribution of aircraft speeds. The FBRP algorithm already allows for flows of aircraft with different speeds.

3. **Climbing and descending traffic and full 3D.** Our study assumes that all flights are at a common altitude, in order that we can determine the throughput associated with any one flight level. Future work will address the full 3D model in which flights may be ascending or descending while passing through or entering/leaving the airspace of interest. In principle, the FBRP routing algorithm can be extended to full 3D routing, using more or less the same method. The theoretical throughput bounds also can be defined in terms of maxflow/mincut theory; however, the mincut problem in 3D becomes challenging algorithmically (and is likely intractable), so further study is necessary to determine efficient methods of approximating the theoretical maximum throughput (e.g., number of air lanes) for the full 3D problem.

4. **Time-varying weather constraints.** Future work will address throughput bounds in the presence of moving and changing weather constraints. The routing of flows in FBRP explicitly models and solves the problem in the presence of dynamic weather constraints, taking as input a sequence of time-sliced weather forecasts. However, the theoretical maxflow/mincut analysis assumes static constraints, and the complications of handling dynamic constraints, e.g., by working in \((x,y,t)\) space as does FBRP, raises some of the same challenges as does the full 3D problem. More generally, of course, the goal is to handle the full 4D model in \((x,y,z,t)\) space.

5. **Stochastic weather models.** In our model we assume that the position and size of the constraints in the airspace are deterministic (i.e., known in advance rather than being random variables). In reality, the weather is estimated via a forecast, or an ensemble of forecasts, with a degree of uncertainty that increases with the time horizon of the forecast. With an ensemble set of forecasts, those forecasts that have a higher level of belief (e.g., based upon historical accuracy or on some prior distribution) are assigned higher probabilities. The ensembles can be generated by perturbed initial conditions, multiple models applied to the same initial conditions, using bred vectors, or through a data assimilation process, as a few examples. A weighted mean of the ensemble may be chosen as the estimated forecast, while a covariance may be computed to signify the spread of the ensemble. In [Mitchell et al., 2006], throughput estimation is considered in an ensemble of forecasts; for each of the forecasts, the mincut is computed using techniques described here for the deterministic case, and then the probability distribution of the mincut value is computed explicitly, along with the mean, variance, etc. In future work, we will extend our results here to stochastic weather models, including ensembles of forecasts and probabilistic weather maps in conjunction with an explicit modeling of the spatial correlation between nearby points.

6. **Human Factors.** In this paper, we have studied the maximum throughput of an airspace limited by geometric constraints. Future work must address how human factors limit the capacity of an airspace, in particular, the workload of the controller or pilot.

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# LIST OF ACRONYMS AND SYMBOLS

2D Two-dimensional
3D Three-dimensional
4D Four-dimensional
A Set of arcs in a graph
\( a_i \) Aircraft \( i \)
AA All Altitudes Rule
AAR Alternating Altitude Rule
ATM Air Traffic Management
\( B \) The “Bottom” portion of the boundary of an airspace, separating source and sink edges
\( c(e) \) Capacity of arc \( e \) in a graph
\( C(p,t) \) Estimated complexity around point \( p \) at time \( t \)
\( d(a_i,p) \) Distance from the \( i^{th} \) aircraft \( a_i \) to \( p \)
dBZ Decibels Z; measure of radar reflectivity for weather
\( e \) An arc
\( f \) A flow
\( f(e) \) Flow through arc \( e \)
FBRP Flow-Based Route Planning/Planner
FCA Flow Constrained Area
\( G \) A directed graph
\( H_i \) The \( i^{th} \) polygonal region of hazardous weather
\( l_{ij} \) The length of the edge connecting \( i \) and \( j \) in the critical graph
MAP Monitor Alert Parameter
MIT Miles-In-Trail
MR Monotonic Rule
\( N \) Set of nodes in a graph
\( N(p,t) \) Density of aircraft around point \( p \) at time \( t \)
NGATS Next Generation Air Transportation System
NWS National Weather Service
\( P \) The polygonal boundaries of the airspace of interest
\( p \) An arbitrary grid point in the complexity calculations
\( R \) The radius of interference in the complexity calculations
RNP Required Navigational Performance
\( S \) A set of nodes in a network graph containing the source
\( s \) Source node in a network, or the arrival/source edge of a polygonal airspace
\( s_i(p) \) Scaling factor for aircraft \( a_i \)’s contribution at point \( p \) in the complexity calculations
\( T \) A set of nodes in a network graph containing the sink, or the “Top” portion of the boundary of an airspace, separating source and sink edges
\( t \) Sink node in a network, or the departure/sink edge of a polygonal airspace
UR Unidirectional Rule
\( V_{avg}(p,t) \) Local average velocity of neighboring aircraft around point \( p \) at time \( t \)
\( v_i \) Velocity of the \( i^{th} \) aircraft \( a_i \)
\( Var(p,t) \) Variance in aircraft velocities around point \( p \) at time \( t \)
WSI Weather Severity Index
BIBLIOGRAPHY


**BIOGRAPHIES**

**Jimmy Krozel** is a Senior Engineer in the Research and Analysis Department at Metron Aviation. Jimmy Krozel received an AS (1984, Computer Science), BS (1985, Aeronautical Engineering), MS (1988, Aeronautical Engineering), and Ph.D. (1992, Aeronautical Engineering) from Purdue University. Krozel was a Howard Hughes Doctoral Fellow (1987-1992) while at the Hughes Research Labs (1987-1992). Krozel is an Associate Fellow of the AIAA, has over 60 technical publications, and is the winner of two AIAA best paper awards. His research interests include computational geometry, visualization, ATM, air traffic control, intelligent path prediction, intent inference, and autonomous vehicles.

**Joseph S. B. Mitchell** received a BS (1981, Physics and Applied Mathematics), and MS (1981, Mathematics) from Carnegie-Mellon University, and Ph.D. (1986, Operations Research) from Stanford University. Mitchell was with Hughes Research Labs (1981-1986) and then on the faculty of Cornell University (1986-1991). He now serves as Professor of Applied Mathematics and Statistics and Research Professor of Computer Science at Stony Brook University. His primary research area is computational geometry, applied to problems in ATM, sensor networks, computer graphics, visualization, manufacturing, and geographic information systems.

**Joseph Prete** is a Senior Analyst in the Research and Analysis Department at Metron Aviation. Prete received a BS (1999, Computer Science) from Polytechnic University and a Ph.D. (2007, Computer Science) from Stony Brook University. His principal area of research is the application of computational geometry to ATM problems.

**Valentin Polishchuk** received a diploma (1996, Applied Physics and Mathematics) from Moscow Institute of Physics and Technology, and an MS (2002, Operations Research) and a Ph.D. (2007, Applied Mathematics and Statistics) from Stony Brook University. Polishchuk worked as a researcher in the Environmental Modeling Lab of the Nuclear Safety Institute, Moscow (1998-2000), and as a research assistant in the Applied Mathematics and Statistics Department at Stony Brook (2002-2007). His main research area is algorithmic motion planning with applications in ATM, sensor networks, and robotics. He is with the Helsinki Institute for Information Technology as a postdoctoral researcher starting in 2007.