Light scattering and ink penetration effects on tone reproduction

Li Yang, Reiner Lenz, and Björn Kruse
Institute of Science and Technology, Linköping University, S-601 74, Norrköping, Sweden

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Light scattering, or the so-called Yule–Nielsen effect, and ink penetration into the substrate paper play important roles in tone reproduction. We develop a framework in which the influences of both of these effects on the reflectance and tristimulus values of a halftone sample are investigated. The properties of the paper and the ink and their bilateral interaction can be parameterized by the reflectance $R_p$ of the substrate paper, the transmittance $T_i$ of the ink layer, the parameter $\gamma$ describing the ink penetration, and $\beta$ describing the Yule–Nielsen effect. We derive explicit expressions that relate the reflectance of the ink dots ($R_i$), the paper ($R_p$) and the halftone image ($R$) as functions of these parameters. We also describe the optical dot gain as a function of these parameters. We further demonstrate that ink penetration leads to a decrease in optical dot gain and that scattering in the paper results in the printed image’s being viewed as more saturated in color.

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1. INTRODUCTION

It is well known that the optical and rheological properties of ink and substrate paper and their mutual interaction determine the color appearance of the reproduced image. Attempts to explain the properties of the printed image in terms of these characteristics led in the late 1930’s to the Murray–Davis formula and the Neugebauer equations. The Murray–Davis formula [Eq. (1)] describes the reflectance $R$ of the image as a whole as a linear combination of the reflectance $R_i$ of the ink dots and the reflectance $R_p$ of the bare paper. The weights are the size of the dot and the area of the paper, given by $f$ and $1 - f$, respectively:

$$R = R_i f + R_p (1 - f).$$

(1)

The Neugebauer equations are generalizations for color prints and are given by

$$W = W_i f + W_p (1 - f) \quad (W = X, Y, Z)$$

(2)

where $W_i$ and $W_p$ are the tristimulus values of the ink and the paper. Although the Murray–Davis formula and the Neugebauer equations are conceptually correct, it soon became clear that they give inaccurate descriptions of experimental results. In 1951 Yule and Nielsen interpreted the discrepancy as the result of light penetration and scattering in the paper substrate. This is now known as the Yule–Nielsen effect. This led to a modification of the Murray–Davis formula, which was replaced by

$$R = [R_i^{1/n} f + R_p^{1/n} (1 - f)]^{n}.$$

(3)

A similar change was also adopted for the Neugebauer equations (see Ref. 4). The exponent $n$ is usually obtained by fitting the experimental data (such as optical density). This provides a numerical approximation but no physical insight into the real process. In 1978 Ruchdeschel and Hauser obtained an estimate of the exponent $n$ in terms of the point-spread function (PSF), describing the scattering of light in paper. The study showed that $1 < n < 2, but there is also experimental evidence that there are many exceptions for which $n \geq 2$. $R$ can therefore vary when $f$ has an exponent higher than 2 (i.e., $f^n$ with $n > 2$). In addition, both theoretical and experimental studies have pointed out that the exponent $n$ itself may depend on $f$, especially in the case where $f > 50\%$.

Recently the Yule–Nielsen effect was studied further through the use of numerical, probability, and analytical methods. Gustavsson investigated the Yule–Nielsen effect by direct numerical simulation of the scattering events that depend on the optical properties of the materials, the halftone frequency, and the halftone geometry. Based on a PSF approach, Rogers presented a method dealing with the light-scattering process in the case of no ink penetration and proposed a matrix approach in which the tristimulus values of a halftone image are calculated as the trace of a product of two matrices. In these studies the effect of ink penetration into the substrate was barely touched, and no explicit expression of the tristimulus values as functions of dot percentage $f$ (as in the Neugebauer equations) has been given. Arney et al. extended the probability model that was originally introduced by Huntsman to take into account the effects of optical dot gain and ink penetration. This approach provides some insight at little computational cost. However, as noticed by the authors themselves, the simple model of ink penetration overestimates the optical effect of scattering, which sometimes leads to results that do not match expectations.

Recently we established a theoretical approach to account for the effect of ink penetration in which we assume that the substrate paper has been uniformly covered by ink layers. In the present paper we generalize this approach by investigating halftone images for which light
scattering is important. In the next section we describe a framework that includes both light scattering (Yule–Nielsen effect) and ink penetration. In Section 3 we further explore these effects and derive expressions for the reflectance and optical dot gain (and the corresponding tristimulus values) as functions of dot coverage. Moreover, the approach is illustrated with some examples. Finally, we summarize the results.

2. MODEL AND METHODOLOGY

The basic geometry used here is shown in Fig. 1, where the surface of the substrate paper has been divided into two sets: \( \Sigma_1 \), the paper under the dots (or ink-penetrated paper), and \( \Sigma_2 \), the bare paper (or paper between dots). For simplicity, only the case of a single layer of dots is analyzed. The extension to a multilayer case is straightforward but tedious and therefore is not given. Also for simplicity, we assume that the ink layer has a uniform thickness [see Fig. 2(b)].

**A. Point-Spread-Function Approach**

Let us first examine the process of light reflection from a microscopic point of view [see Fig. 2(b)]. We consider several separate steps. First, we assume that \( r_1 \) is an arbitrary position on the surface of the paper under the dot (\( \Sigma_1 \)) and that \( r_2 \) is an arbitrary position on the surface of the paper between the dots (\( \Sigma_2 \)). Now consider an element light source \( I_0 \, \text{d} \sigma_1 \) that strikes the dot at \( r_1 \). The flux of light detected at \( r_2 \), due to scattering of the incident light from \( r_1 \), may be written as

\[
d^2J_{12} = p(r_1, r_2) T_{12} I_0 \, \text{d} \sigma_1 \, \text{d} \sigma_2.
\]

The transmittance of the ink is \( T_{12} \), and \( T_{12} I_0 \, \text{d} \sigma_1 \) is thus the amount of light entering the substrate under the dot. The PSF \( p(r_1, r_2) \) is the probability of photons entering the dot at position \( r_1 \) and exiting from the paper at position \( r_2 \). It is worth noting that ink penetration destroys the uniformity of the substrate. Assumption \( p(r_1, r_2) = p(|r_1 - r_2|) \), which was generally applied in non-ink-penetration analyses, becomes invalid. Next, we consider an extended light source covering the area of dot coverage and keep the intensity of the incident light \( I_0 \) unchanged. The paper between the dots is not illuminated. Then the flux of light detected at \( r_2 \), due to scattering of the incident light from dots, is the integration of Eq. (4) over the dot's area (\( \Sigma_1 \))

\[
dJ_{12} = I_0 T_{12} \int_{\Sigma_1} p(r_1, r_2) \, \text{d} \sigma_1 \, \text{d} \sigma_2.
\]

Therefore \( \int_{\Sigma} p(r_1, r_2) \, \text{d} \sigma_1 \), is the probability that the incident light that enters the substrate under the dots will be scattered into position \( r_2 \). Finally, by performing the integration over the whole area of the paper between dots (\( \Sigma_2 \)), one obtains the total amount of light detected at the paper between dots that is scattered from the light incident on the dots:

\[
J_{12} = I_0 T_{12} \int_{\Sigma_1} \int_{\Sigma_2} p(r_1, r_2) \, \text{d} \sigma_1 \, \text{d} \sigma_2.
\]

The double integral

\[
\int_{\Sigma_1} \int_{\Sigma_2} p(r_1, r_2) \, \text{d} \sigma_1 \, \text{d} \sigma_2
\]

is the overall probability of photons' being scattered from \( \Sigma_1 \) into \( \Sigma_2 \), which is therefore a measure of the Yule–Nielsen effect. In the case of no ink penetration, a general expression of the integral was carried out by Rogers. In the final step we exchange the position of the lighting source with that of the detector; for example, in the first step we put the light source \( I_0 \, \text{d} \sigma_1 \) at \( r_2 \), put the detector at \( r_1 \), and keep other conditions unchanged. Then we have

\[
d^2J_{21} = p(r_2, r_1) T_{21} I_0 \, \text{d} \sigma_1 \, \text{d} \sigma_2.
\]

From the optical reciprocity one obtains the following relation:
\( p(r_1, r_2) = p(r_2, r_1). \) \hspace{1cm} (8)

Then we have

\[ J_{12} = J_{21}. \] \hspace{1cm} (9)

This equation means that under the uniform illumination of the halftone sample the amount of light being scattered from \( \Sigma_1 \) (halftone dots) into \( \Sigma_2 \) (bare paper) is equal to the light scattered from \( \Sigma_2 \) to \( \Sigma_1 \). The calculation of the flux \( J_{12} \) requires the knowledge of the PSF, which is usually not available, especially in the case of existing ink penetration. However, if the mean value \( \bar{p} \) of the integrated PSF defined as

\[ \bar{p} = \frac{1}{f(1-f)} \int_{\Sigma_1} \int_{\Sigma_2} p(r_1, r_2) d\sigma_1 d\sigma_2, \] \hspace{1cm} (10)

is available, then the scattered light \( J_{12} \) can then be calculated as

\[ J_{21} = J_{12} = I_0 T_i \bar{p} f(1-f). \] \hspace{1cm} (11)

Evidently \( \bar{p} \) depends not only on the physical properties of the substrate paper and the ink but also on the geometric and spatial distribution of the dots. As is shown below, \( \bar{p} \) is closely related to optical dot gain and is therefore experimentally measurable.

**B. Probability Approach**

Now we study the process of light reflection from a macroscopic viewpoint. First, we assume that there is no ink left on the surface of the substrate paper (see Fig. 2a); i.e., the system consists of paper that is partially penetrated by the ink. If a photon strikes the substrate in \( \Sigma_1 \), we define conditional probabilities \( P_{11} \) and \( P_{12} \) that the photon will reemerge from \( \Sigma_1 \) (ray 1a), and \( \Sigma_2 \) (ray 1b), respectively. Similarly, for a photon entering the substrate in \( \Sigma_2 \), \( P_{21} \) and \( P_{22} \) are defined as the probabilities that the photon will leave the surface of the substrate from \( \Sigma_1 \) and \( \Sigma_2 \), respectively. These probabilities fulfill the following constraint conditions:

\[ P_{11} + P_{12} = \alpha \] \hspace{1cm} (12)

\[ P_{21} + P_{22} = \beta, \] \hspace{1cm} (13)

where \( \alpha \) and \( \beta \) are constants depending on the optical properties of the materials and three-dimensional geometric and spatial distribution of the ink inside the substrate paper. The values of \( \alpha \) and \( \beta \) are less than 1 if light is absorbed or transmitted at the other side of the medium. Intuitively, \( \alpha \) and \( \beta \) are related to the reflectance of the system, which describes the probability of a photon’s striking the surface of the substrate from the air and then returning to the air.

If the percentages of ink-penetrated paper and the bare paper are \( f \) and \( (1-f) \), respectively, and if the intensity of irradiance onto the whole system is \( I_0 \), the flux of photons striking the \( \Sigma_1 \) and \( \Sigma_2 \) areas are \( I_0 f \) and \( I_0 (1-f) \), respectively. Then the flux \( J_{mn}^0 \) of photons from \( \Sigma_m \) to \( \Sigma_n (m, n = 1, 2) \) is given by

\[ J_{11}^0 = I_0 f P_{11}, \]

\[ J_{12}^0 = I_0 f P_{12}, \] \hspace{1cm} (14)

\[ J_{21}^0 = I_0 (1-f) P_{21}, \]

\[ J_{22}^0 = I_0 (1-f) P_{22}. \] \hspace{1cm} (15)

The total flux of photons emerging from the bare paper \( J_p^0 \) is therefore a summation of \( J_{12}^0 \) and \( J_{22}^0 \), i.e.,

\[ J_p^0 = I_0 [P_{12} f + P_{22} (1-f)]. \] \hspace{1cm} (16)

The reflectance of the bare paper can thus be calculated as

\[ R_p^0 = \frac{J_p^0}{(1-f) I_0} = P_{12} \frac{f}{(1-f)} + P_{22}. \] \hspace{1cm} (17)

Similarly, the total flux of photons exits from \( \Sigma_1 \) (ink-penetrated paper), and its corresponding reflectance reads

\[ J_{11}^0 = I_0 [P_{11} f + P_{21} (1-f)], \] \hspace{1cm} (18)

\[ R_i^0 = P_{11} + P_{21} (1-f)/f. \] \hspace{1cm} (19)

Since \( J_{12}^0 \) and \( J_{21}^0 \) (Eqs. (14) and (15)) are special cases of \( J_{12} \) and \( J_{21} \) with \( T_i = 1 \), one obtains from Eq. (11) the following expressions:

\[ P_{21} = \bar{p} f, \] \hspace{1cm} (20)

\[ P_{12} = \bar{p} (1-f). \] \hspace{1cm} (21)

These expressions provide a symmetric expression of \( P_{21}, P_{12}, \) the dots, and the paper between the dots: The probability of photons going from one part (paper or dot) to the other is proportional to the area of the arrival part. Evidently \( P_{12} \) and \( P_{21} \) are related by:

\[ P_{12} f = P_{21} (1-f). \] \hspace{1cm} (22)

Thus we have [from Eqs. (12), (13), (17), (19), and (22)]

\[ R_i^0 = \alpha, \] \hspace{1cm} (23)

\[ R_p^0 = \beta. \] \hspace{1cm} (24)

In the case of an ink layer existing on the substrate surface [see Fig. 2(b)], \( \Sigma_1 \) is the area under the dots, and \( \Sigma_2 \) is the paper between the dots (i.e., the bare paper). We now treat the ink layer as a filter with transmittance \( T_i \), and therefore we can use the definitions and results obtained so far. We especially see that \( J_{11} = T_i J_{11}^0, J_{12} = T_i J_{12}^0, J_{21} = T_i J_{21}^0, \) and \( J_{22} = J_{22}^0 \). For the uniformly illuminated halftone image with light of intensity \( I_0 \) the reflectances of the printed ink dots and of the paper between the dots are given by

\[ R_p = \frac{J_{12} + J_{22}}{(1-f) I_0} = P_{22} + P_{12} \frac{f}{1-f} T_i, \] \hspace{1cm} (25)

\[ R_i = \frac{J_{11} + J_{21}}{f I_0} = P_{11} T_i^0 + P_{21} \frac{1-f}{f} T_i. \] \hspace{1cm} (26)

Applying Eqs. (23) and (24) and using the expressions for \( P_{12} \) and \( P_{21} \) [Eqs. (20) and (21)], one can express the reflectance in terms of the optical properties of the material involved as

\[ R_p = R_p^0 - \bar{p} f(1-T_i), \] \hspace{1cm} (27)

\[ R_i = T_i [\gamma R_p^0 T_i + \bar{p} (1-f)(1-T_i)]. \] \hspace{1cm} (28)
The quantity \( \gamma \) describes the effect of ink penetration on the reflectance of the substrate paper. Because of strong absorption from the ink-penetrated paper, \( \gamma \) is generally less than 1. Equations (27) and (28) describe the dependence of the reflectance on the optical properties of the materials (variables \( T_i \) and \( R_{\text{p}}^0 \)), ink penetration (\( \gamma \)), and light scattering in the media (\( \rho \)). The quantity \( \gamma \) depends only on the distribution of the penetrating ink which was investigated recently.\(^\text{16}\) The value of \( \rho \) depends on the size and the spatial distribution of the printed dots, the ink penetration, and the optical properties of the materials involved. Thus the knowledge of \( \rho \) is of critical importance in predicting and reproducing the desired reflectance. The variable \( R_i \), however, does not depend on \( \gamma \), and therefore \( R \) and \( R_i \) should be used when parameters must be fitted to experimental data.

The quantity \( \gamma \) is an important parameter in the model, which may be measured indirectly. Suppose that the measured reflectance of a printed patch (with solid ink coverage) is \( R_i \). The thickness of the pure ink layer can be measured by a microscope (or more advanced microscopic imaging instrument); the thickness can be converted into the transmittance of the ink layer \( T \) according to an exponential law. Therefore \( \gamma \) can be computed as \( \gamma = R_i / (T^2 R_{\text{p}}^0) \), where \( R_{\text{p}}^0 \) is the reflectance of the bare paper.

3. DISCUSSION AND EXAMPLES

Equations (27) and (28) describe how reflectance properties depend on material properties and geometry. They reveal two important facts:

1. \( R_p \) and \( R_i \) are no longer constants, as they were assumed to be in the Murray–Davis equation, as soon as light scattering is taken into account. This result is in line with previous work by Arney and his colleagues.\(^\text{17}\)

2. In the case in which \( \rho \) is independent of the dot coverage \( f \), \( R_i \) and \( R_p \) vary linearly with \( f \). In the other words, the nonlinearity of \( R_p \) and \( R_i \) with respect to \( f \) provides information about the \( f \)-dependence of the light scattering effect (or \( \rho \)).

A. Reflectance of a Halftone Image and Optical Dot Gain

The reflectance of the halftone sample is given by

\[
R = R_i f + R_p (1 - f),
\]

which is a quadratic function of \( f \) if \( \frac{d\rho}{df} = 0 \), as can be seen from Eqs. (27) and (28). For \( \frac{d\rho}{df} \neq 0 \), this is no longer the case, and the relation between \( R \) and \( f \) depends on the form of \( \rho \).

Substituting the expressions of \( R_p \) and \( R_i \) [Eqs. (27) and (28)] into Eq. (30), one gets

\[
R = R_{\text{MD}} - \Delta R,
\]

where

\[
R_{\text{MD}} = R_i^0 T_i^2 f + R_p^0 (1 - f)
\]

is the reflectance of the halftone sample without light scattering (i.e., the Murray–Davis value). We write \( R_{\text{MD}}(f) \) and \( R(f) \) if we consider the values \( R_{\text{MD}} \) and \( R \) as functions of \( f \). Light scattering inside the substrate paper is described by

\[
\Delta R = (1 - T) \rho f(1 - f).
\]

Because \( \Delta R > 0 \), the true reflectance \( R \) is smaller than its Murray–Davis value \( R_{\text{MD}} \), and the halftone image appears to have larger dot coverage than predicted when scattering is ignored. This is why this effect is known as optical dot gain. If scattering is not modeled, then the measured reflectance \( R \) seems to originate from a dot size \( f + \Delta f \) instead of the true dot size \( f \). From \( R(f) = R_{\text{MD}}(f + \Delta f) \) one can then obtain the optical dot gain \( \Delta f \) as the function of the optical properties of the materials and the ink penetration:

\[
\Delta f = \frac{\Delta R}{R_p^0 (1 - \gamma T_i^2)} = \frac{(1 - T) \rho f(1 - f)}{R_p^0 (1 - \gamma T_i^2)}.
\]

The optical dot gain is thus proportional to \( \rho \). From the measured optical dot gain profile, one can therefore estimate \( \rho \) and obtain valuable information about the PSF.

The maximum of the optical dot gain can be obtained from the equation

\[
\rho' f(1 - f) + \rho(1 - 2 f) = 0,
\]

where

\[
\rho' = \frac{d\rho}{df}.
\]

For \( \rho' = 0 \) or \( \rho = \text{constant}(\neq 0) \), the optical gain has a single maximum at \( f = 50\% \) and has a symmetric form around the maximum.

We now illustrate the influence of the form of \( \rho \) on the reflectance functions and the optical dot gain. In the first example we consider \( \rho = R_p^0 \). In the case of no ink penetration (\( \gamma = 1 \)), this corresponds to the Yule–Nielsen model with the Yule–Nielsen exponent \( n = 2 \), which corresponds to so-called complete scattering.\(^\text{8}\) In this case [see Fig. 3(a)] the reflectances \( R_i \) and \( R_p \) vary linearly with the dot area \( f \). The mean reflectance \( R \) of the whole image, on the other hand, varies in a quadratic fashion. The calculated optical dot gain has a parabolic profile with a single maximum at \( f = 50\% \).

In the second example we adopt

\[
\rho = R_p^0 [1 - f^m (1 - f)^{1 - m}],
\]

with \( m = 0.7 \). Now all reflectance values, especially \( R_p \) and \( R_i \) [see Eqs. (27) and (28)], are highly nonlinear functions of the dot coverage [see Fig. 4(a)]. Their behavior is characteristic for typical AM halftone images, as can be seen from the measurements.\(^\text{12}\) The sharp decrease of \( R_p \) as \( f \) increases shows that the probability \( P_{22} \) of a photon’s entering and then exiting from the bare paper decreases dramatically when the area of the bare paper becomes relatively much smaller than the area of the ink dots. In other words, the probability \( P_{22} \) increases dramatically as the ink coverage increases. Accordingly, the maximum of the computed optical dot gain has been shifted downwards to \( f = 37\% \).
The dependence of the reflectance values on the ink penetration has also been shown in Fig. 4, which we will explore further in the next subsection. We use these two examples only to illustrate the power and flexibility of the model developed above. They do not simulate any real print. In a real application one can either determine \( \bar{\rho} \) by fitting the measured data (such as an optical dot gain profile) or compute it by integrating the PSF over the halftone pattern, using Eq. (10).

**B. Optical Effects of Ink Penetration**

We assume that the thickness and transmittance of the printed ink layer are \( d \) and \( T_0 \), respectively, and that the reflectance of the paper is \( R_p^0 \). The penetration of a part of the printed ink into the substrate paper has two optical effects. It is known that the absorption comes mainly from ink. When there is no ink penetration, the ink layer is relatively thicker; the ink becomes thinner after part of it penetrates the substrate paper. Therefore (for the latter case) there is less light absorption when the light goes through the ink layer, i.e., \( T_i^0 > T_i^p \). A cancellation effect comes from the weakness of the reflectance of the substrate paper due to presence of penetrating ink, i.e., \( R_i^p < R_i^0 \). However, owing to the high scattering power of the substrate paper (by the paper fiber), light probably does not go through all the penetrating ink when it is reflected back. The expected combined effect is either that there is less light absorbed on average or that there is stronger light reflection, i.e., \( T_i^0 R_i^p < T_i^p R_i^0 \). Numerical simulation on ink penetration, which covered a wide range of ink and paper (i.e., a wide range of absorption and scattering properties of ink and paper) has confirmed this expectation.\(^{16}\)

The following simulations illustrate the influence of ink penetration on the properties of the halftone image. Here we computed the reflectance values \( R_p, R_i, \) and \( R \) and the optical dot gain \( \Delta_f \), with and without considering the effect of ink penetration. In the case of ink penetration, we have assumed that \( T_i = 1.3T_0 \) and \( \gamma = R_i^0/R_p^0 = 0.8 \). As is shown in Fig. 4, all the reflectance values increase when ink penetration is taken into account (dotted curves). The largest relative increase is obtained for

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Fig. 3. (a) Computed reflectance and (b) optical dot gain with \( \bar{\rho} = R_p^0 = 0.87, T_i = 0.2 \).

Fig. 4. (a) Computed reflectance and (b) optical dot gain with \( \bar{\rho} \) given in Eq. (35), where \( R_p^0 = 0.87, m = 0.7 \). Solid curves, no ink penetration, \( T_i = T_0 = 0.2, \gamma = 1 \); dotted curves, with consideration of ink penetration, \( T_i = 1.3T_0, \gamma = 0.8 \).
the reflectance of the ink dots \( R_i \). Owing to light scattering, the ink penetration also changes the reflectance of the bare paper, \( R_p \). Ink penetration also decreases the optical dot gain. It is possible that the absorption of the ink by the paper decreases the probability of photon exchange between the areas \( \Sigma_1 \) and \( \Sigma_2 \). This dependence of the optical dot gain profile on the ink penetration could be used to obtain estimates of material properties such as \( \bar{p} \) and \( \gamma \).

**C. Investigation of Color Printing**

Finally, we sketch how the methodology developed above can be used in an investigation of color ink printing processes. As usual, we compute tristimulus values by integrating the product of the reflectivity \( R(\lambda) \) (here we explicitly denote its dependence on the wavelength of the illuminated light), the illumination \( S(\lambda) \), and the tristimulus functions \( \bar{x}(\lambda) \), \( \bar{y}(\lambda) \), and \( \bar{z}(\lambda) \). To simplify the notation, we use \( \bar{w}(\lambda) \) to represent one of the tristimulus functions \( \bar{x} \), \( \bar{y} \), or \( \bar{z} \). The results of the integration are the tristimulus values \( X, Y, \) and \( Z \), and we use \( W \) to represent any one of them. As in Eq. (30), we get the tristimulus value \( W \) of the halftone sample

\[
W = W_f + W_p(1 - f),
\]

where

\[
W = \int R(\lambda)S(\lambda)\bar{w}(\lambda) d\lambda,
\]

\[
W_i = \int R_i(\lambda)S(\lambda)\bar{w}(\lambda) d\lambda,
\]

\[
W_p = \int R_p(\lambda)S(\lambda)\bar{w}(\lambda) d\lambda.
\]

Despite the similar structure of Eq. (36) and the original Neugebauer equations, they are rather different. Both \( W_p \) and \( W_i \) depend on \( f \), since both \( R_p \) and \( R_i \) depend on the dot area. Therefore the linearity assumption underlying the original Neugebauer equations does not hold any longer. Moreover, \( W_p \) is not just a function of the color coordinates of the paper, since scattering from the ink area has to be taken into account.

Following the approach used in Eq. (31), the influence of the light scattering on the tristimulus values can be described by

\[
W = W_{MD} - \Delta W,
\]

where

\[
W_{MD} = \int R_{MD}(\lambda)S(\lambda)\bar{w}(\lambda) d\lambda
\]

(38)

is the contribution from light following the Murray–Davis assumption, and

\[
\Delta W = \int \Delta R(\lambda)S(\lambda)\bar{w}(\lambda) d\lambda
\]

(39)

corresponds to the Yule–Nielsen effect. From Eq. (37) and the nonnegativity of \( \Delta W \), we find that for any tristimulus value \( W_0 \) we have

\[
W_0 - W \geq W_0 - W_{MD}.
\]

4. **CONCLUSION**

We derived a new model for analyzing the properties of tone reproduction. We concentrated mainly on the investigation of the effects of light scattering and ink penetration on the final print. For this we derived a new model in which we described the reflectance properties of the print by three types of parameters:

1. The ink penetration [given by \( \gamma \) defined in Eq. (29)],
2. The transmittance of the ink layer \( T_i \) and the reflectance of the bare paper \( R_g \), and
3. The effect of light scattering described by \( \bar{p} \).

We showed that incorporation of the light scattering effect results in reflectance functions that are highly nonlinear functions of the dot area. We also derived a de-
scription of the optical dot gain as a function of these parameters and illustrated how the properties of the scattering function \( \bar{p} \) influence the optical dot gain. We demonstrated further that the model predicts that ink penetration leads to a decrease in optical dot gain and that scattering in the paper results in the printed image’s being viewed as more saturated in color.

The relationship between the optical dot gain \( D_f \) and the scattering function \( \bar{p} \) [Eqs. (10) and (33)] can be used in both ways; for known (or defined) scattering (\( \bar{p} \)) the relationship can be used to predict the optical dot gain. For measured optical dot gain profiles, on the other hand, this data can be used to obtain estimates of the scattering properties as described by \( \bar{p} \).

The analysis was not restricted to any specific type of halftoning scheme; therefore, all of the expressions and conclusions apply to both AM and FM halftone schemes.

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