SHAPE GRAMMARS
SIGGRAPH 2009 Course

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The theory of shape grammars, first launched by Stiny and Gips in 1972, defines a formalism to support the ambiguity in creative processes that is generally ruled out by quantitative and symbolic computations. Since then, it has evolved into a groundbreaking pragmatist philosophy of shape and design. It is implemented in fields varying from architecture, art, graphic design, industrial product design to computer visualization. This course offers basic knowledge on the theory and some advanced issues useful for its implementation.

The course will be in two consecutive sessions which are introductory and advanced and last 1 ¾ hrs each. The two-partite introductory lecture presents the fundamentals of the theory, focusing on the basic knowledge of shapes, shape algebras, and shape rules in order to explain how shape grammars translate visual and spatial thinking into design computation. Examples of shape grammar applications in design analysis and synthesis will be presented. Attendees with further and more technical interest in the topic are encouraged to follow the advanced lecture which initially dwells on the computational devices of shape grammars then to discuss a number of selected studies on the computational implementation of the shape grammar idea.

**Prerequisites:** No prerequisites for the first session other than enthusiasm for shapes and a keen interest in looking and seeing. For the second session, general knowledge of the theory of shape grammars, which can be acquired in the first session.
SHAPE GRAMMARS
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SYLLABUS

Session 1 – Introduction
8:30 – The theory
1. What are shape grammars?
2. Describing shape grammars in terms of seeing and counting
3. Describing shape grammars as a rule-based system
4. Decompositions
5. The mathematical set-up of shape grammars
6. Basic elements: shapes, labels, weights
7. Shape algebras
8. Shape boundaries
9. Part relations: embedding, overlapping, discrete elements
10. Euclidean transformations
11. Maximal shapes
12. Boolean operations on shapes

Break

9:30 – What to do with it?
13. The arts
14. Cultural heritage
15. Procedural modeling of architecture
16. Mass-customized housing
17. Economy of architectural manufacturing
18. Classifying architectural form
19. Building brand identity
20. Movement grammar in interaction design
21. Arts and Crafts
22. Understanding design possibilities (in design education)

Close, Q&A

Session 2 – Advanced Issues
10:30 – Recursion, Identity, Embedding
1. Recursion is the key to calculating.
2. Units create designs with blind ease.
3. Recursion and identity go just so far.
4. Embedding alters everything.
5. Seeing never ends.

Break

11:30 – Recursion, Identity, Embedding continued
7. Hierarchies are seductive.
8. The value of embedding is that units and hierarchies never get in the way.
9. Recursion plus embedding includes recursion plus identity.

Close, Q&A
(1) It is such a delight to be talking about shape grammars at this convention. And it is a great challenge. Not only because the audience is diverse, but also because the shape grammar theory itself dwells on something that is quite obvious but we take for granted. With my students back at home, I sometimes manage to talk so convincingly about the theory that they say “we knew that!” But my real aim is to get them start thinking about how they use it. We could aim for something similar here. Some of the things I say will seem quite mundane, but I encourage the audience to think whether they use or not use these in what they do, or how they think about their work.
Part I – Introduction to the Theory

(2) Title page to Session I
What are shape grammars?

<table>
<thead>
<tr>
<th>a) A computation theory</th>
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<tbody>
<tr>
<td>that defines a formalism to represent visual (and spatial) thinking;</td>
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<tr>
<td>that handles <strong>ambiguities</strong> which symbols do away with.</td>
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<tr>
<th>b) A philosophy of looking at the world</th>
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<td>that is not through learnt or imposed definitions but through those that have a <strong>practical meaning</strong> at a given point in time;</td>
</tr>
<tr>
<td>that values the continuity of matter and flexibility in how to cut it up into its parts.</td>
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</table>

(3) Shape grammars may be described at two levels. Firstly, it is a computation theory that defines a formalism to represent visual, or even spatial, thinking. At the same time, it handles ambiguities which digital computing does away with. Off the center, marginal,

The phrase shape grammar more literally refers to visual design grammars.

At the second level, the theory represents a philosophy of looking at the world that is not through learnt or imposed decompositions (definitions) but through those that have a practical meaning at that point in time.
What are shape grammars?

(4) Shape grammars were first introduced in the beginning of the 70s by George Stiny and James Gips. Published as one of the best computer papers of 1971, their “Shape Grammars and Generative Specification” paper introduced a set of generative rules for a few paintings done by Stiny himself.

The three paintings in the article, are from a series called Urform. These are going to be the basis for illustrating various concepts of shape grammars in this part of the lecture.
What are shape grammars?

(5) Stiny (2006) claims that design is calculating while expanding the meaning of calculation to visual thinking via his theory of shape grammars. The motto “design is calculating,” was a starting point in 1971 as well. The reasoning behind a visual product was described using a grammar-like formalism with a vocabulary, a set of rules, and a series of computations that produced designs as if they were “sentences.”
(6) Stiny often equates the terms design, visual reasoning and calculation. This claim firstly enunciates an understanding that design has reasoning within. Secondly, in the theory of shape grammars, the terms calculation and computation, which are often interchangeably used, are seen under a new light.

It is important to reflect on seeing and counting simultaneously to understand the key idea in SG.

I have put up a graphic of the abacus to represent counting, which is at the root of computing and calculating. Beads, discrete and of one kind, are counted. Counting is one aspect of reasoning.

Visual calculation on the other hand, gives room for seeing as well as for counting.
How does one calculate with shapes?

_Urform II, George Stiny, 1970, acrylic on canvas 30 ins x 57 ins, blue, red, orange, yellow._

(7) Questions arise. How does one calculate with shapes? Do visual kinds of thinking exclude calculation? Or does calculation reduced to counting exclude visual and spatial kinds of thinking? Stiny argues that one has to really ‘see’ in order to count and that ‘seeing’ is where creativity lies.
Seeing and Counting

Counting requires discrete parts.

(8) As in the abacus, counting requires discrete parts.
(9) One can divide Urform II into smallest possible discrete bits, perhaps into dots on the screen, each assigned with a different color code. This image shows a small section of the imagined screen of dots.

These smallest primitives are countable but irrelevant in the perception of the whole.
Seeing and Counting

But the painting is not simply the sum of discrete parts known beforehand.

(10) Alternatively, one can divide *Urform II* into some obvious parts, distinct therefore countable. There are two of ... However, the painting is possibly a much more dynamically formed formal arrangement and is not simply a sum of discrete parts that were known before hand.
(11) There are always some other parts to see. Moreover, these may be the meaningful parts, or parts that are surprisingly merged with one another. In the visual world, there are wholes that coexist, and they share parts, or parts of parts. This image shows a part that is not readily there but can be seen.
(12) Calculation then, is to see first, then count. Key idea. What we take for granted is seeing. This way, we can calculate with different parts each time we look at Urform II. The shape shown exists in ten instances in Urform II: one large, nine small ones.
Seeing and Counting

Varying parts and wholes coexist.

(13)
Seeing and Counting

Parts and wholes coincide.

(14)
The shape grammar way of seeing and counting is visual rules that tell: “see the left side and then replace it with what is on the right.”

(15) Stiny and Gip’s explanation for the process behind the Urform series is a visual rule that tells one to see the left side to replace it with the right side. The illustration shows one such possible rule. These kinds of rules form the basis of shape grammars.
Seeing and Counting

I can see two instances of the shape on the left side of the rule.

(16) This is how it basically works. Looking for the left side in an initial shape set, in this case Urform I, one can see two instances of it.
I apply my rule to one of them.

(17) The second one is rotated 180°.
(18) The rule is applied to the second one shown.
(19) The rule is then applied to the first instance.
Shape grammars is a rule-based formalism. Rules show the particular shapes to be replaced and the manner in which they are replaced. The marker shows how to align the two shapes. Rather than “if A, then B,” visual rules say “see SHAPE$_1$, do SHAPE$_2$."

(20) Shape grammars is a rule-based formalism. This aspect is picked up more easily. Applications...

A shape rule has two steps when applied: a recognition of a particular shape shown on the left side and its possible replacement shown on the right side.

The defined rule is operational. The arrow indicates an action. The unique feature of a shape rule is that the left and right side are visually considered. As opposed to symbols, shapes can be looked at and seen differently. This is due to their inherent ambiguities.
Because shapes are visual, they can be decomposed in infinitely many ways. There should be no preconceived decompositions and primitives acquired through such operations. Visual rules, which are subjective, will call for various decompositions.

For example, let us look at one of the most popular examples Stiny (2006) gives to explain why we need to be computing with visual rules. There is a shape, composed of three triangles that will be rotated around its center. The only catch is, it will be rotated by a rule that says “rotate triangle.”
Useful Decompositions

(22)
Useful Decompositions

The visual rule: rotate an equilateral triangle 180° around its center.

(23)
Useful Decompositions

Stiny’s nine-step computation where the initial shape of “three triangles” is redefined as “two triangles” at steps 4 and 6.

(24) In the nine step computation, Stiny shows that the initial definition of the shape, that is ‘three triangles’, changes in step 4 and then back again in step 6. Decompositions should not be timeless. The initial shape could have been drawn as three triangles, six lines, or 9 lines. Whatever the history, a new definition can always come up while working with shapes. What you see is what you get. This is motivation to see more. Ambiguity should be maintained.

Any questions?
(25) The mathematical set-up of the theory includes general definitions of shapes, shape, weight and label algebras, shape boundaries, the most important of all part relations, Euclidean transformations, maximal shapes, and Boolean operations with shapes.
Shapes, labels, weights

Basic elements of shapes are points, lines, planes, and solids, with **labels**, if necessary, to give abstract information about them, and **weights**, as indicators of magnitudes of some formal attributes.

(26) Shapes can be points, lines, planes, solids or combinations of these. Shapes also can have labels that indicate additional information about them and weights that indicate the magnitude of some formal properties. Labels are useful for adding more constraints necessary for tasks such as establishing the order in which rules are applied in computations.
Shapes are categorized under different shape algebras. The left index shows the dimension of the basic elements, and the right index shows the dimension in which these basic elements are combined in shapes.

(27) Basic elements in shapes are categorized under different shape algebras. The indices indicate the dimension of the basic element and the dimension of the space in which these elements are combined and transformed.
All shape algebras that have 0 for the first index are atomic. A basic element within these algebras can only be a point and has no parts other than itself. Beads on the abacus belong here. Symbols (even if visual), for example, are elements of these algebras and have a dimension of zero. Also, units that add up to a sum of units belong in these algebras but in those that have the second index higher than 1.
(29) The algebra where both indices are 0 is Boolean. There are only two values, null and one. Something either is or is not.
(30) All algebras with the indices equal to or larger than one, show different properties than atomic algebras. They do not have atoms but shapes with parts such as lines, planes, solids, etc. The number of members within a set in one of those algebras does not have to be finite. For example, in algebra $U_{11}$, on a line space, there can be infinitely many lines of different lengths.
(31) There is a clear relation between the categories of basic elements belonging to different algebras. The boundaries of solids are plane shapes, the boundaries of planes are line shapes, the boundaries of lines are points whereas points have no boundary. Number of parts is finite in point algebras, in others no... hence the ambiguities.
Shape boundaries

$U_{12}$ and $U_{22}$ algebras are combined when utilizing the relation between shapes and shapes on their boundaries.

(32) Shape boundaries constitute a practical relation between shapes, which, in turn, helps us in the way we visually think.

The rule in the illustration is in $U_{12}+U_{22}$ algebras combined. Parts of plane boundaries appear as line shapes and are utilized in generating the final form with planes.
Part relations

Three types of part relations are those of

- overlapping,
- embedded, or
- discrete shapes

(33) Part relations are what differentiates shapes from atoms. Three kinds of part relations are between overlapping, embedding and discrete shapes.
(34) Planes with no shared boundaries are discrete.
(35) Shapes that share a common boundary but have no part in common are also discrete.
(36) The two planes highlighted in slides 36 and 37 share a common boundary, but share no plane parts.
(37) Thus they are discrete despite the common boundary.
(38) Those shapes that share a common part overlap.
(39) The two planes shown share a common part, and are overlapping. Both shapes have parts that are not common with the other.
(40) Those shapes that share a common part overlap.
Part relations

(41) If two shapes have common parts and at least one of these shapes has no part that is not a part of the other, then this shape is said to be embedded within the other. The darker shape is embedded within the larger and lighter colored shape.
Euclidean transformations

<table>
<thead>
<tr>
<th>Rotation</th>
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<tbody>
<tr>
<td>Translation</td>
</tr>
<tr>
<td>Mirror reflection</td>
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<tr>
<td>Scaling</td>
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...and combinations of these

(42) Euclidean transformations that are used in shape grammars are translation, scaling, rotating and reflecting along with their combinations. In the example of the painting, I can relocate the left side of the rule that I showed in so many places using these transformations. I can scale it down and up, I can see its rotations, I can see its reflections, and I can see it in multiple places, which are illustrated in slides 44 through 48.
(43) Let us start with any perceived shape within *Urform II.*
Euclidean transformations

scaling

(44) I can identify it in a smaller size.
Euclidean transformations

reflection

(45) I can identify it in a mirror reflection.
(46) I can identify it in a 90° counter clock wise rotation.
(47) I can identify it in another location in the painting.
Boolean operations on shapes

<table>
<thead>
<tr>
<th>Operation</th>
<th>Formula</th>
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<tbody>
<tr>
<td>Sum</td>
<td>$A + B$</td>
</tr>
<tr>
<td>Difference</td>
<td>$A - B$</td>
</tr>
<tr>
<td>Product</td>
<td>$A \cdot B = A - (A - B)$</td>
</tr>
</tbody>
</table>
| Symmetric difference | $A \Box B = (A - B) + (B - A)$  
                     $A \Box B = (A + B) - (A \cdot B)$ |

(48) Within the defined shape algebras, we can add and subtract shapes of the same kind of basic elements. We can also take their unions and products. This is basically how we compute the visual rules.

We can combine algebras to do Boolean operations on different kinds of basic elements in parallel.
(49) Here are illustrations to possible Boolean operations on shapes based on the Urform series. The first operation shows the symmetric difference of two plane shapes of the same weight in $U_{22}$ whereas the second operation shows the sum of the boundaries of these two planes in $U_{12}$. 
Let us assume that there are three initial shapes for another set of examples of operations on shapes of equal weight value in $\mathbb{U}_{22}$. 
(51) Firstly, the difference of shapes one and two is calculated.
(52) Then, the sum of shapes two and three is calculated and...
Boolean operations on shapes

\[ \text{\textbullet} \quad \text{\textbullet} \quad \text{\textbullet} \quad \text{\textbullet} \quad \text{\textbullet} \quad \text{\textbullet} \]

(53) … subtracted from the result of the first step.
(54) Continuing with the operations, the product of shapes two and three …
Boolean operations on shapes

(55) … is followed by the symmetric difference of the two.
(56) The three shapes, newly emerged from these operations, are assigned different weights and summed up.

Up until this point, we have shown how shape algebras, Boolean operations and part relations all work separately for computing with shapes. In the next part, more examples, from actual applications, will be utilized to illustrate these concepts further and together.
(57) Break
What to do with it?

(1) In this section, we will dwell on a selected set of existing and possible applications of the theory in various design related areas. In addition to architecture, where it is most popular, there are quite a few different venues that shape grammars are applied in, from crafting to brand identity, and from interaction design to urban design. Examples usually are categorized as analysis, synthesis or a combination of both approaches in design, all useful for different needs. Additionally, examples can be viewed according to whether they primarily make use of the rule-based approach (i.e. description of design decisions with rules and the design process as computations with these rules) which is straight forward to understand and apply from a systematic perspective, or of the unique formalism that allows for part relation (embedding) in the application of rules (i.e. computation without the need for predefined primitives and open to surprises). Looking at the upcoming examples, I would like to encourage you to engage in seeing parts as much as in understanding the recurring rules.
The arts


(2) Exploring formal constructions in art are directly relevant to the explorations that the theory of shape grammars dwells on and encourages. Ilhan Koman, a Turkish artist who has lived and produced mostly in Sweden, has dedicated his life to the systematic study of simple geometric forms and the variety attained from their derivatives. He stands for a conscious artist persona whose work is teaching to many art and design students on the pragmatic relation of art with mathematics and geometry. His works are mostly cases of recurring spatial relations that transform. The Rolling Lady is the display of two instances of a particular geometric shape spatially connected to one another. The PI series explore the different degrees in which a spatial relationship can be applied and that its recursions lead to various results. Although Koman is not quoted for having utilized shape grammars, his works showcase the theory from within the field.

(3) Russell Kirsch is not only known as the creator of the first digital image but also as one of the earlier people who embraced the idea of a rule-based picture grammar. In a study dating back to 1988, Kirsch and Kirsch analyze and define a grammar for Diebenkorn’s Ocean Park painting series.
(4) Focusing on the “fluidity of the artist’s mark” on the canvas and attributing due value to seeing shapes as they are, Jacquelyn Martino identifies a curvilinear shape grammar in analysing her own art work and process.

The art work presented here to sample the style is an early digital painting in the *Devotion series*. On the right are three rules that show early phases of development of another work of similar process.
(5) Understanding, preserving, continuing, reinterpreting and progressing processes behind forms of cultural heritage is an important and developing application field for shape grammar studies. These studies may employ both the analysis of design corpus and synthesis of new designs if relevant to the context. Additionally, developing computer aids for such analysis and synthesis is of great practical value. In this example, we see a study that not only analyses a traditional craft of cloth patterns, namely Kuba cloth, but also introduces the notion of “intelligent tracing” of such forms and their spatial relations using computer visualization tools built in Mathematica. The illustrations are of a sample Kuba cloth (on the left), initial modular shapes (in the middle), and the Kuba grammar rules with left and right hand sides (on the right).
(6) Islamic patterns have been of interest to the Shape Grammars community for some time. The studies so far dwell on the recursion of modules or tiles under Euclidean transformations. The continuity, perhaps the most important theological concept employed in these patterns, actually might be calling for a more thorough analysis of varying parts and wholes in perception. These patterns are not only works to be admired on facades of historic buildings or interior artifacts of various material, but also systems of lines that showcase geometric construction to many design students. Therefore the study of the process of how they are constructed, not as tesselation of tiles according to one scholar argument, may be quite relevant in synthesis of new ones.
(7) Ömür Bakirer has observed in ancient documents that the patterns are constructed based on regular tesselations of circles. This is a different approach than identifying tiles that repeat. Lines are continuous and present different parts and wholes to the eye.
(8) The basic visual rule to build the tesselation of circles is that each new circle is drawn with reference to an existing circle passing through its center, and at the same time centering on its perimeter. If this rule application is not narrowed, tesselations can be as varied as the group shown below the rule.
(9) Nonetheless, circles are deterministically arranged, and overlaying grids of new lines are constructed.
(10) New lines are added as groups of them start making up the shapes of the final pattern.
(11) Alternatively new circle tessellations with circles of varying sizes can be introduced.
(12) In the abundance of what one can see, various polygons can also come forth.
(13) The polygon in the previous slide is actually from an existing example carved in stone. One can identify different repeating tiles, stars or polygons each time one looks. Wholes keep changing to the eye. To approach these constructions as tilings could be an underestimation in most cases.
(14) There are more complex examples where tiles are not easy to read at all!
(15) A recent and most celebrated work on quasi-crystalline Islamic tilings highlight the understanding of these patterns as tessellations of predefined units. The end product however still allows for different readings of parts and wholes.
(16) These patterns also exist in 3D and as structural architectural elements and not just decoration. In muqarnas, units are pre-cut and carefully placed to form a continuous inverted cascade.
(17) In the infamous Topkapi Scroll, a documentation of the repertory of geometric designs dating back to the 15th and 16th centuries, we see that the geometric relations between units are pre-studied and documented in drawings that reveal the geometric understanding behind them to guide the craftsmen in the manufacturing of these designs.

Source: Gülru Necipoğlu, 1995, The Topkapi Scroll—Geometry and Ornament in Islamic Architecture, Getty Center for the History of Art and the Humanities, Santa Monica, CA.
(18) Since most muqarnas are architectural elements that enclose spaces, full understanding of their construction and possible synthesis of new designs may be extremely relevant in restituting or restoring ruins. The photograph shows what is left from a muqarnas in an Armenian church as an example to suggest that the knowledge of its grammar might help in completing its missing parts.
(19) Continuing on the thread of cultural heritage, another group of examples are from modeling of architecture or urban environments. The procedural modeling of Pompeii, as shown here, is one of the results of a study that may contribute to understanding and appreciating historic built environments. The study, in fact, has broader implications from building facades to building city structures such as roads, or constructing virtual gaming environments. The models are based on context sensitive shape grammar rules.

For more information and examples see http://www.vision.ee.ethz.ch/~pmueller/wiki/CityEngine/Documents
Lipp, Wonka and Wimmer take the approach of procedural modeling to the next level and allow for interaction to edit rulebases visually rather than through the script.
Mass-customized housing


(21) One of the well known applications of the theory of shape grammars in architecture is the Siza grammars developed by Jose Duarte for mass customizing social housing by a world renown Portugese architect. The project comprises of an analysis of Siza’s Malagueira housing design corpus, developing its detailed shape grammar that Siza himself is content about, and the synthesis of new designs in which the users are actively involved through a computer interface.
(22) In architecture, mass housing is a significant issue especially in developing countries. Shape grammars that can help designers enumerate customized alternatives without much cost are crucial in sustaining desired built environments that meet standards at the least. Sotirios Kotsopoulos has also dwelled on this notion in his entry for a Habitat for Humanity housing competition with a proposed shape grammar.
(23) Parametric rules for general massing of units, and modularity were key elements of the proposal.
(24) Botha and Sass introduce mass customization in housing with elevated concern for economy and environmental conditions. The grammars they utilize allow for adaptation in designs based on changing conditions.
Economy of architectural manufacturing

FIGURE


(25) Their approach also addressed fast and transportable housing production needs in natural disaster emergency and poverty stricken locations.
Economy of architectural manufacturing

(26) Sass’s approach overall aims to integrate design synthesis based on changing needs with digital fabrication for a low cost but customized production in the end.

(27) The wood frame grammar that is the construction system in these examples was developed by Sass. The tables show, in part, the rule set of joints or how larger parts come together (on the left) and the classification of building component types and their relations (on the right).
(28) Another application of the theory in relation with architecture is the attempt to classify general architectural forms through a grammar based on a set of common properties (of changing values).
(29) In an application in the industrial design field, shape grammars are utilized to showcase an established brand identity for Buick cars. In the figure above a sample of novel Buicks are shown. The variety is created to address specific needs or desires.
(30) Asokan and Cagan introduce the “movement grammar” for actions of coffee drinking in a specific culture. They perform and analysis of movement rituals to form a grammar, and use this grammar in the design of objects that are directly utilized in the said actions. They address the unique notion of movement grammars, cultural languages and interaction design simultaneously.
(31) To go back to the simple 2D shapes that we started with in the beginning of the section, let us go back in time as well. The interest in how shapes are constructed out of recurring parts or how they are decomposed into unprecedented parts has existed for a long time. In *A Theory of Pure Design*, Ross, Harvard professor, shows, in as early as 1907, how to construct various shapes out of parts. Source: Denman W. Ross, 1907, *A Theory of Pure Design: Harmony, Balance, Rhythm*, Houghton, Mifflin and Company, Boston and New York, p 25, 40, 41, 46, and 65. The first shows varying distance in the spatial relation of points, the second shows a symmetric group of mirror-reflected curvilinear parts, the third, fourth, and fifth show transformations of arcs compiled in groups to bring about continuous forms. Ross utilized this grammar and likes of it in creating wallpaper patterns as the one shown.
(32) One of Ross’s contemporaries, the Mexican artist Best Maugard, introduced a basic vocabulary of shapes that Daniel Kornhauser puts to use in his research on craft computing. Best Maugard writes, “The suggestions and rules that we will follow are simple and easily understood by everyone. They are quickly grasped and retained in the mind of the student. In this method, there are seven simple motifs and signs, which we consider as fundamental, and a few rules to follow, and these, once in the student’s memory, will enable him to make an infinite number of combinations and designs…” See Adolfo Best-Maugard, 1926, A Method for Creative Design, Alfred A. Knopf, New York and London, p 1-2. He aims to identify a finite global vocabulary of basic elements and sees design as combinatorial arrangement of these elements. His vocabulary is quite strict and limited compared to Ross’s. Nonetheless, Kornhauser understands the value and utilizes this vocabulary to develop spatial relations and rules that result in designs ready to be crafted.
(33) Kornhauser shows the design and manufacturing process for a copper plate, from the digital tool to the hand crafting.
(34) Kornhauser illustrates, through numerous screenshots, the digital crafting of a spider web design.
(35) The theory can also be utilized to introduce beginning design students to an understanding of the design process in which decisions are traced, questioned, exploited to the full extent of possibilities. In a very simple formal organization exercise, the top left figure is an actual proposal to a given problem that asked for the arrangement of 9 identical units in a square format. In a scenario where possible spatial relations of two units are tried first (the visual rule given above), the complex internal arrangement of the unit is reduced down to one line. Eight transformed instances of this unit is given in the middle row. This reduced version already provides many possibilities to try out. On the right, the alternate black and white shadings are also introduced increasing the possibilities. The three layouts below left, showcase different arrangements of just these pairings in a group of 9 units.

That the theory of shape grammars could be applied in design education to demystify design processes in the eyes of the novice designer is perhaps among its most valuable traits towards societies that are more design oriented.
The End of the Introduction
Part II – Recursion, Identity, Embedding

Title page to Session II
Recursion is the key to calculating in the way Turing and others recommend, and a staple today in logic, linguistics, and computer science – in fact, wherever calculating is tried. The way recursion works is clear when rules are used to change arrangements of independent units or symbols in a combinatory process. In this example, a rule made up of squares inscribes a small square in a big one. The square in the left side of the rule occurs twice in the right side to provide another place for the rule to apply again.
(2) The Russian constructivist Jacob Tchéïnikhov uses the rule for squares and another rule just like it for quadrilaterals in these designs in black and white.
(3) Rules and recursion define Chinese window grilles. These lattice designs are called ice-rays.
(4) The rules for ice-rays divide polygons into polygons. The division rule \( x \rightarrow \text{div}(x) \) and the addition rule \( x \rightarrow x' + x'' \) are equivalent, when \( \text{div}(x) \) divides \( x \) into \( x' \) and \( x'' \). The polygons \( x, x', \) and \( x'' \) are always triangles, quadrilaterals, or pentagons.
(5) The rules apply in this way to create an ice-ray lattice.
This ice-ray was shown at SIGGRAPH 2008 in the first SIGGRAPH exhibit on design and computation.
(7) Some ice-rays are produced in a definite way.
(8) Rules are applied from left to right. It’s the same in action painting when up strokes and down strokes alternate across a surface.
(9) Rules make smaller divisions to add finer detail.
(10) Multiple divisions are also possible with tri-axial motifs, and motifs with four, five, and six axes.
(11) In these ice-rays, there’s an initial division with a multi-axial motif.
(12) This ice-ray is also from the SIGGRAPH 2008 exhibit.
(13) It’s easy to define rules for symmetrical ice-rays and ones with other special properties.
(14) This ice-ray lattice is created almost entirely with tri-axial divisions.
(15) The ice-ray is one of my favorites, because its divisions are so novel.
(16) Ice-rays modulate light and cast changing shadows.
(17) The recursive division/addition rules for ice-rays can be used in many other ways, too. For example, the top figure is a painting by Georges Vantongerloo, and the bottom figure is a plan by the architect Alvaro Siza. Both have perpendicular divisions.
(18) This is a painting by Fritz Glarner. Angles vary, but not by much.
These plans show the two floors in a medieval building in Venice.
The ground floor has perpendicular divisions, and the upper floor has parallel divisions. The plans look different because of this.
Current systems are not only remarkably inflexible, but tend to hang on to ontological commitments more than is necessary. Thus consider this sequence of computer drawings. Suppose that the figure in step 2 was created by first drawing a square, then duplicating it, as suggested in step 1, and then placing the second square so as to superimpose its left edge on the right edge of the first one. If you or I were to draw this, we could coherently say: now let us take out the middle vertical line, and leave a rectangle with a 2:1 aspect ratio, as suggested in step 3. But only recently have we begun to know how to build systems that support these kinds of multiple perspectives on a single situation (even multiple perspectives of much the same kind, let alone perspectives in different, or even incommensurable, conceptual schemes).

(21) Recursion may not be all there is to calculating when it comes to visual experience. This is the problem shape grammars solved more than thirty years ago. The key is to use recursion with embedding instead of identity – to calculate with shapes by seeing and not with units or symbols by counting. This may go beyond what Turing originally had in mind. It asks what calculating would be like if Turing had been an artist/designer and not a logician. Embedding alters everything. One shape is embedded in another shape if it can be cut out or traced. Units aren’t defined in advance, because there’s no telling what they are ahead of calculating. They change freely as rules are applied. This isn’t so with identity. Units are defined once and for all at the start, in some sort of precalculating before rules are tried. Units stay the same. In combination, they determine (limit) what there is to see and how to go on. But this contrast may be misleading. Calculating isn’t a dichotomy: at the very least, calculating by seeing includes calculating by counting, because identity is a special case of embedding.
These designs are created recursively by combining triangles. They're the units. The design \( f(n + 1) \) is four copies of the design \( f(n) \), so triangles are times four. If there's a single triangle to start, \( f(n) = 4^n \). But visual experience may disagree. How many triangles are in \( f(2) \)? There are \( 4^2 = 16 \) and 12 more. And that's not the half of it – what about the two squares in \( f(1) \), and the cross and \( 2 \times 2 \) grid in \( f(2) \)? These new figures may be easier to see than triangles. They really stand out. Can I calculate with them if triangles disappear? Where do the triangles go?
(23) It’s true that \( f(3) = 64 \), but what does \( f(3) \) look like? Do you see any triangles? How many? Four copies of \( f(2) \) are in \( f(3) \). Do the figures in \( f(2) \) stand out? In general, \( 4m \) copies of \( f(n) \) are in \( f(n+m) \). Are these copies or the figures in them obvious? Aren’t there other things to see? Is anything salient now a combination of triangles or anything you’ve seen before? It’s easy to combine units in designs, but the process is blind – it misses the variability in visual experience. Recursion and identity go just so far. They’re visually incomplete. Embedding fills in the rest, so that you can calculate with anything you see.
(24) There are two squares.
(25) Two squares are lots of things – four triangles, and also pentagons and hexagons in various ways.
(26) A rule that translates polygons also rotates them! Polygons are embedded in surprising ways that may ignore what you’ve done. There’s no record of this to block your way, and nothing to remember. Anything you see can be used to go on.
A polygon is a closed plane figure with \( n \) sides.

(27) How are polygons defined? *Wikipedia* gives a definition and helpful examples.
(28) Maybe these figures are polygons, too – pentagons and heptagons.
(29) Then there are K’s – big ones and little ones – like those in my dictionary.
(30) Big K’s come in any size.
(31) There are just as many little k’s.
Whether it's for a big K or a little k, the series is dense. There are myriads to see.
(33) Seeing makes a difference. Both of these building plans (diagrams) are based on two squares, but they’re articulated in alternative ways to express different things. The plans may even be opposites. One plan uses four K brackets to define exterior walls and interior corners, while the other plan uses four triangular pieces for exterior corners and interior walls.
(34) This is a lesson that’s taught in the third grade. It’s a plan or map of a room. The schoolchildren draw it and describe what they find. There’s seeing and saying.
(35) This is the key for the objects in the room. It’s a list of rules to see what’s there. Are these rules shapes or symbols? Do they apply in terms of embedding or identity? The lesson is a nice way to introduce such questions, although educators may not know it. (This may be one of the many times in school when children are expected to give up their fickle ways and learn an adult answer, that is to say, to trade embedding for identity (creativity for greater certainty) as rules are tried. Then it seems that the purpose of education is to limit what there is to see and take what little remains seriously. The loss of ambiguity and breadth may make communication and shared understanding easier – some value to the community is undeniable – but the cost is too high if you aren’t free to look again. Calculating with shapes and rules – seeing – doesn’t work by rote. It’s an open-ended process that’s independent of what you’ve done or may remember. What you see may change erratically at any time. It’s always a surprise!)
(36) How many tables are in the drawing? How many desks? How can you tell? What do you see? What did my daughter say when she was asked to count tables and desks? How did I reply? What answer did the teacher expect? Are there other ways to do this? When do embedding and identity agree?

<table>
<thead>
<tr>
<th></th>
<th>Tables</th>
<th>Desks</th>
<th>Rules</th>
</tr>
</thead>
<tbody>
<tr>
<td>Teacher</td>
<td>2</td>
<td>1</td>
<td>Symbols (identity)</td>
</tr>
<tr>
<td>Catherine</td>
<td>3</td>
<td>1</td>
<td>Shapes (embedding + isometry)</td>
</tr>
<tr>
<td>Dad</td>
<td>5</td>
<td>4</td>
<td>Shapes (embedding + similarity)</td>
</tr>
</tbody>
</table>
(37) This is another kind of Chinese window grille based on a checkerboard lattice. The squares in a grid are filled with H's that go this way or that on alternating diagonals.
(38) Here are more examples of the same kind of lattice design.
(39) These are the motifs that are used in the squares in the underlying grid. They alternate horizontally and vertically, and may be easier to find than H's.
(40) What do you see? Is this merely a checkerboard?
(41) A vector field goes through the lattice from left to right. It’s for everyone to see.
Maybe the forces are in equilibrium.
(43) No, it’s an illusion! What a neat way to go from physics to a trick of perception. With embedding, you can change your mind about what you see every time you look.
(44) Seeing never ends. Look again.
(45) It may help to erase the grid to create a new lattice design.
(46) The lattice is an array of squares.
(47) The lattice is a tessellation of Greek crosses in a figure-ground reversal.
(48) Is this a checkerboard lattice?
(49) The grid is rotated 45°, and a new motif is oriented left or right on diagonals. This is an effective way to create a checkerboard lattice, but there’s usually more to calculate once the lattice is done. It’s easy to go on with embedding.
(50) What do you see? Maybe octagons, octagon-squares, and supplementary squares – but surely, not a checkerboard. The grid and the motif have disappeared.
(51) This is a Palladian villa plan. It uses the same kinds of rules that are used for Chinese window lattices. The same rules can do all sorts of different things.
(52) This partial catalog shows 20 possible plans.
(53) All of these plans were created by the rules. Palladio designed some of them. Can you guess which ones? Even the experts are fooled. The confusion is telling!
(54) A rule to translate polygons rotates them when rules are defined with embedding. There are other ways to calculate like this, too, in the same family of shapes.
A rule that rotates triangles about their centers keeps these points fixed. But the rule also rotates a pinwheel, so that these points change. This can’t be right. Is it a new paradox? It isn’t something to think about, it’s something to see.
(56) This is the way the rule is used to calculate. Three triangles are two, and two triangles are three when there’s embedding. Isn’t this cheating?
Hierarchies are a seductive way to show how things work by dividing them into independent parts and mapping their relationships. Hierarchies take work – they make your brain hum – and they reward the effort with an aura of understanding. Many find hierarchies indispensible. They make things memorable and comprehensible. Rules also define hierarchies when they apply recursively. But this may be a clumsy and disappointing way to describe what’s going on with embedding, because then things change. Parts aren’t independent – they fuse and divide freely, with scant regard for what they were before. Nonetheless, comparing alternative hierarchies may help to show how complicated it can be to turn seeing into counting when you calculate.
An erasing rule defines rival hierarchies for the same shape. There are three triangles or two in the way I’ve been calculating.
(59) This is how the triangles in the hierarchies are related. What these triangles have in common divides them retrospectively to define finer units that are consistent with both hierarchies and that augment them.
This is how to change one hierarchy into another. The graphs are isomorphic – the one shows the switch from three triangles to two, while the other goes from two triangles to three. It may get complicated when units are moved around. There are apt to be knots and tangles. But this kind of thinking is tedious and unnecessary. It’s much easier to see three triangles or two, to switch what you see whenever you like without worrying about what units there are and where they go. That’s the value of embedding. Units and hierarchies never get in the way. They aren’t needed to calculate, and they aren’t needed to see.
Here’s another example with triangles and squares.
Four triangles are two squares, and vice versa. With embedding, it’s easy to switch back and forth at any time. But with identity, this isn’t as straightforward. It’s hard to find the units that allow for this kind of change, especially before calculating begins. How do you know that triangles are going to be squares or anything else – maybe pentagons, hexagons, or K’s? You may need the prior ability to calculate with shapes and embedding in order to define the units you need to calculate with identity – either that, or a special kind of prescience. Or maybe calculating with embedding is merely pseudo-calculating. You do it to learn how to calculate. But then, what’s the point of calculating if you already know the answer? Perhaps it’s to save time and effort doing the same kind of problem again. Ice-rays are like that, and so are checkerboard lattices. Of course, creating these designs is only a start. There’s always something new to see. In art and design, it’s hard just to do it again and not to see and do more.
(63) These are the hierarchies for four triangles and two squares.
(64) This is the way the triangles and squares in the hierarchies match up to define finer units to add to the hierarchies.
(65) This is the way the hierarchies are changed one into the other. It's a little more complicated than it was before with three triangles and two, but the pattern is clear.
The two previous examples open an ongoing series of triangles, triangles and squares, triangles and pentagons, etc.
(67) The shapes in the series can be elaborated with rules. The number of different ways of seeing the shapes in each row in terms of triangles, triangles and squares, triangles and pentagons, etc. grows exponentially – the number of ways for the n-th shape is the n-th Fibonacci number, when Fibonacci numbers go 1, 2, 3, … . Of course, there are other polygons in the shapes, and many other surprises, too, that may pop in and out of view. There’s no dismissing any of this without serious risk. It’s impossible to tell in advance what will be of use or when it might be needed.
(68) These are some of the simple rules I've been using with embedding to calculate with shapes.
Twin lattices order various types of rules for art and design. Whatever creativity implies is possible when recursion and embedding are used together to calculate. For example, it’s easy to do everything on the fly, to change what you see as you go on in a visual kind of improvisation.
(70) Paul Klee did this drawing – it’s a “palm-leaf umbrella.” But it looks more like a fan. Try using the rules in the first lattice to make it. Is this easier to do with the rules in the second lattice?
(71) Recursion plus embedding (calculating by seeing in the artist’s/designer’s way) includes recursion plus identity (calculating by counting in Turing’s way). Calculating by seeing extends calculating by counting. The inverse of this relationship is something to think about, too. Can recursion plus identity do (simulate) everything that recursion plus embedding does? This is an open question worth trying. But perhaps there’s no complete answer, just many ad hoc ones. For example, the answer is yes for shapes made up of points, lines, planes, and solids in the algebras $U_{ij}$, and their extensions with labels (numbers, symbols, etc.) and weights (colors, materials, properties, etc.) in the algebras $V_{ij}$ and $W_{ij}$. This is a good start, and, in fact, it covers a lot. If you need more, there are also affirmative answers for conics, and cubic curves and surfaces. Or better yet, add to this list!
(72) Don’t forget Klee’s drawing! Try the rules – they really work.
The End

(73) Break
SHAPE GRAMMARS
SIGGRAPH 2009 Course

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**Suggested readings**


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