1 Introduction

The use of radial basis functions (RBFs) has a long history in computer graphics. Typical applications of RBFs are continuous interpolation/approximation of discrete data, and field functions of metaball-like implicit surfaces. In this paper we introduce eccentric radial basis functions (ERBFs) in order to enhance the power of expression of RBFs. An ERBF is an RBF equipped with a point called "eccentric center", which is used to compute the distance for the RBF and introduce eccentricity into the distribution of the function values; the gradient of the function becomes gentle at the far side of the eccentric center while steep at the near side of it. The extension is simple yet powerful, and allows us to fit data with fewer basis functions than RBFs, especially when fitting or modeling data consisting of both gentle and steep gradations. We demonstrate the effectiveness in applications such as fitting of scattered data and modeling with ERBF-based metaballs.

2 Eccentric Radial Basis Functions (ERBFs)

The concept of ERBFs was originally invented by Eiji Takaoki early in 1985 and used in his art work in 1992 (which is found in [Bloomenthal and Wyvill 1997]) for enhancing field functions of metaballs to represent steeply curved surfaces. He referred to such metaballs as "eccentric metaballs. We conclude that these results have demonstrated the power of expression of ERBFs.

![Figure 1: Calculations of r for (a) an RBF and (b) an ERBF.](image1)

An RBF \( \phi(r) \) is a function of the distance \( r \) from the center \( c \) to a point \( x \) in \( d \)-dimension (Fig. 1a), and when a cutoff radius \( R \) is used, \( \phi(r) = 0 \) in the range of \( r > R \). Well-known examples of RBFs are Gaussian \( \alpha \exp(-\beta r^2) \) (where \( \alpha \) and \( \beta \) are constants), and a thin-plate spline \( r^2 \log |r| \). While both an RBF and an ERBF are functions of the distance \( r \), they differ in how to compute the distance. For an ERBF (Fig. 1b), \( r \) is measured not from the center \( c \), but from the eccentric center \( e \), and scaled as follows:

\[
\begin{align*}
    r &= \frac{\|e-x\|}{\|e-x'\|} R = \frac{\|e-c, x-e\| + \sqrt{D}}{R^2 - \|e-c\|^2} R, \\
    D &= (e-c, x-e)^2 + \|x-e\|^2 \left( R^2 - \|e-c\|^2 \right),
\end{align*}
\]

where \( x' \) is the intersection point between a \( d \)-dimensional sphere with radius \( R \) and a half line originating at \( e \) and passing through \( x \). Intuitively, by regarding Fig. 1 as the top views of upside-down cones, the distance \( r \) for an RBF is considered as the height of a cone whose apex is located at \( e \). By contrast, for an ERBF, \( r \) is considered as the height when the apex is translated to \( e \).

3 Results and Conclusion

![Figure 2: Approximating a log plot of the Mie phase function (green) using eccentric Gaussians (red). The fitted curve is blue.](image2)

Fig. 2 shows a result of approximating complex data using eccentric Gaussians. As an example, we chose a log plot of the Mie phase function, which is essential to the computation of scattering of lights in computer graphics. We can approximate the data using 10 eccentric Gaussians with 4.6% relative RMS error, while in our experiments we need 17 ordinary Gaussians with 8.0% relative RMS error.

![Figure 3: Rendering results of Takaoki’s torso model using eccentric metaballs.](image3)

Fig. 3 shows rendering results of Takaoki’s torso model consisting of only 24 eccentric metaballs. Note that the breast and the hip exhibit steeply curved surfaces that are hard to represent with ordinary metaballs. We conclude that these results have demonstrated the power of expression of ERBFs.

References