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Critical aspects for learning in an electric circuit theory course – an example of applying learning theory and design-based educational research in developing engineering education.

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ABSTRACT

Transient response is considered to be one of the more difficult parts of learning electric circuit theory, because the mathematics is advanced and the Laplace transform is used to solve differential equations. Usually, the mathematics is taught in math courses and in problem solving sessions, the graphs in lab courses and the conceptual understanding of the transients in lectures, and students are expected to link these components themselves. The idea of conceptual labs has been developed previously for physics courses. In this kind of labs as well physical phenomena as their mathematical and graphical representations are elaborated. In this paper we describe an innovative course in electric circuit theory and how, by introducing systematic changes in lab instruction, it is possible to make students understand the relationship between theory and real circuits.

Keywords: Engineering education research, labwork, electric circuits, educational design, variation theory.

I. INTRODUCTION

In engineering education it is considered important to learn electric circuit theory. For an engineer it is not sufficient to know only DC-circuit theory, since most engineering applications concern time-dependent electrical signals. Students specialising in, for example electrical engineering, control engineering, or engineering physics, are not only required to study AC-circuits, but also the methods for handling complex circuits, various transform methods (Phasor, Fourier and Laplace), and Fourier-series in circuit analysis. Grasping concepts and phenomena from circuit theory, and especially AC-electricity, periodic signals and transients, is important in understanding electronics, telecommunication and control theory.

One of the important aims of an educator is to help students acquire a "functional understanding" of the subject. Marton, Runesson and Tsui [1] have stated this as:

> "Developing a learner's capability of handling novel situations in powerful ways, is considered to be one of the most important educational aims."

Rorty [2] similarly suggests we should not

> "view knowledge as a matter of getting reality right, but as a matter of acquiring habits of action for coping with reality".

In line with this reasoning, Carstensen and Bernhard [3] have suggested:

> "In engineering and in science education one implication of this is that students should *learn* to understand theories and models and their *relation* to objects and events in the 'real' world and *learn* to *apply* these models and theories."

During lab work, students are expected to link observed data to either theoretical models, or to the 'real world' they are exploring. As shown in Figure 1, Tiberghien [4] proposed that the world of theories and models, and the world of objects and events can be seen as main categories in the analysis of knowledge. It is argued that this categorisation is very effective when analysing and developing learning environments, such as labs. According to recent research, students or novices have problems establishing relationships between the object/event world and the theory/model world. It is important to make explicit the links between the theory/model world and the object/event world in education. For example, Vince and Tiberghien [5] state "*establishing relevant relations between the physics model and the observable objects and events is a very difficult task*" and at a physics education conference at Tufts University, the researchers present agreed on the following conclusion [6, 7] (see also [8, 9]): "*Connections among concepts, formal representations, and the real world are often lacking after traditional instruction. Students need repeated practice in interpreting physics formalism and relating it to the real world*" (emphasis in original). Roth [8] found that students were "referentially stuck in the symbolic and associated conceptual representations, and experienced the phenomena as something unrelated".

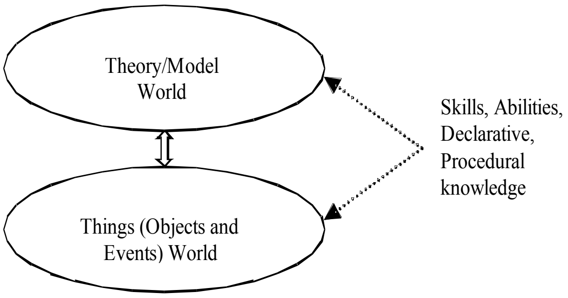


Figure 1. Categorisation of knowledge based on a modelling activity [4].

As mentioned above, one of the aims of lab work is to encourage students to link observed data to either theoretical models, or to the real world they are exploring. Although this is seen as the fundamental purpose of lab work [10], the results from research still show that this aim is rarely fulfilled. The link between the theory/model world and the object/event world has to be made explicit in the lab instructions, otherwise the aim will not be achieved [4, 5].

At university level, the links students are supposed to make between the theory/model world and the object/event world are often links between mathematical models and measurement data, or graphs stemming from mathematical calculations and graphs derived from measurement data. Our research, and studies by other authors, have shown that these links do not occur spontaneously, even when the task is to compare a graph stemming from calculations to graphs derived from measurements.

II. RESEARCH DESIGN AND METHODOLOGY

This study has been developed within a systematic framework to design innovative learning environments in science and engineering education. The aim of our research is to help students acquire a functional understanding of the subject matter. Our work began in 1995 in the field of physics, for engineering students and pre-service teachers, and in subsequent years it has been extended into other domains of science and engineering. Originally, we looked at whether the educational ideas behind innovative curricula, such as RealTime Physics and Tools for scientific thinking [7, 11, 12], could be implemented in a Swedish setting, and if the successful results reported could be replicated. The ideas of experiential learning stemming from Dewey, Lewin and Vygotsky were taken into account in our approach [13-16]. Our work was developed in line with the emergent methodology called “design experiments” [17], “design-based research” [18-20] and “learning/lesson studies” [21, 22], which is different from conventional approaches to designing innovative curricula. As has been reported elsewhere [23-25], our initial efforts were successful. Later this work was extended into other domains including: AC-electricity, involving representations with complex numbers; electrical engineering, involving transient responses and Laplace transforms; and investigations of the critical educational conditions required to encourage insightful learning [26, 27]. A common feature of all these learning environments has been the use of technical artefacts as mediating tools [13, 28-35], enabling, for example, simultaneous displays of many different modes [36] of the concepts involved. In later years we conducted an in-depth analysis of students’ performance in these learning environments.

The aim of this paper is to describe the design of an innovative course in electric circuit theory and how, by introducing systematic changes in the design of tasks, it is possible to develop students’ capabilities to make links between theory and real circuits. We will describe in detail a lab task where the investigation of transient response is the main focus. Transient response is considered as one of the more difficult parts of learning electric circuit theory, which is omitted from many engineering curricula, especially at college level [27].

We have earlier pointed out:

> "What makes [transient response] difficult is that the mathematics used is rather advanced, using the Laplace Transform to solve differential equations. Very often the mathematics is handled in the math course and in the problem solving sessions, the graphs in the lab course and the conceptual understanding of the transients in the lectures, and still it is expected that the students should make links between them". [27]

Below we will describe how our approach is related to design-based educational research and the theory of variation developed by Marton and co-workers.

A. Design-Based Educational Research

In the last ten years, non-conventional approaches to designing innovative curricula have emerged. These approaches have been described as "design experiments" [17] or "design-based research" [18-20]. Cobb et al. [17] described this shift in these terms:

> "Prototypically, design experiments entail both 'engineering' particular forms of learning and systematically studying those forms of learning within the context defined by the means of supporting them. This designed context is subject to test and revision, and the successive iterations that result play a role similar to that of systematic variation in experiment".

The Design-Based Research Collective [20] has described design-based research as having the following five features:

> "First, the central goals of designing learning environments and developing theories or 'prototheories' of learning are intertwined. Second, developments and research take place through continuous cycles of design, enactment, analysis, and redesign … Third, research on designs *must lead to sharable theories* that help communicate relevant implications to practitioners and other educational designers … Fourth, research must account for how designs function in *authentic settings*. It must not only document success or failure but also *focus on interactions* that refine our understanding of the learning issues involved. Fifth, the development of such accounts relies on methods that can document and *connect processes of enactment to outcomes of interest*." (Our emphasis)

Lo et al. [22] have expressed one of the main features of this approach as:

> "The benefits of design experiments are that we will be able to contribute to theory development, and improve practice at the same time."

Our study has several features similar to those described above. We have focused on interactions in an authentic setting. We will demonstrate how we have implemented variation theory [37] in our design and link this to observed outcomes in students' performance. In other papers [38-40] we have discussed how our research has led to the notion of a 'complex concept' – thus our studies have developed sharable theories in addition to reconfirming the value of variation theory.

B. Variation Theory

Marton and co-workers [37, 41, 42] have developed *variation theory,* which is an explanatory framework describing the conditions necessary for learning. Central to this theory is that we learn through the experience of difference, rather than the recognition of similarity. To open up for learning should be understood in terms of *discernment*, *simultaneity* and *variation*. Learning is seen as developing certain capabilities and values that enable the learner to handle novel situations in powerful ways.

Powerful ways of acting emerge from powerful ways of seeing, and our previous experiences affect the way in which we experience a new situation. Our perception also affects the experiences we see as relevant, and the powerfulness of one's act is relative to one's aims in a situation.

> "Thus it can be seen that people act not in relation to situations as such, but in relation to situations as they perceive, experience, and understand them. … If we want learners to develop certain capabilities, we must make it possible for them to develop a certain way of seeing or experiencing. Consequently, arranging for learning implies arranging for developing learners' ways of seeing or experiencing, i.e., developing the eyes through which the world is perceived." [1]

Seeing something in a particular way can be defined by the aspects discerned by a person at a certain point in time. The difference between 'discerning' and 'being told' should be noted.

People discern certain aspects of their environment by experiencing variation. When one aspect of a phenomenon or an event varies, while another aspect or other aspects remain the same, the varying aspect will be discerned. One of the main themes of variation theory is that the pattern of variation inherent in the learning situation is fundamental to the development of certain capabilities. In the words of Marton, Runesson and Tsui [1]):

> "What we believe is that variation enables learners to experience the features that are critical for a particular learning as well as for the development of certain capabilities. In other words, these features must be experienced as dimensions of variation."

According to Marton, Runesson and Tsui [1] the following patterns of variation can be identified:

1. *Contrast*: As mentioned above, in order to experience something, a person must experience something else to compare it to.
2. *Generalisation*: However, in order to fully understand what "three" is, we must also experience varying appearances of "three",
3. *Separation*: In order to experience a certain aspect of something, and in order to separate this aspect from other aspects, it must vary, while other aspects remain invariant.
4. *Fusion*: If there are several critical aspects that the learner has to take into consideration at the same time, they must all be experienced simultaneously.

Experiencing variation amounts to experiencing different instances simultaneously. This simultaneity can either be *diachronic* (experiencing instances that we have encountered at different points in time, *at the same time*) or *synchronic* (experiencing different co-existing aspects of the same thing at the same time).

Marton, Runesson and Tsui [1] also introduce the concept of a learning space:

> "A *space of learning* comprises any number of dimensions of variation and denotes the aspects of a situation, or the phenomena embedded in that situation, that can be discerned

> due to the variation present in the situation. Variation that is not present in the situation can still be discerned, however, if variation is brought in by means of the learner's memory of previous experience. We should notice, here, that 'a space' does not refer to the absence of constraints, but to something actively constituted. *It delimits what can be possibly learned (in sense of discerning) in that particular situation. … The space of learning tells us what it is possible to learn in a certain situation [from the point of a particular object of learning]. … The space of learning ... is ... an experiential space.* …" (First emphasis in original.)

Marton and his co-workers [1, 37, 43] make the distinction between the *intended object of learning*, the *enacted object of learning* and the *lived object of learning*. The intended object of learning is the subject matter that the teacher or the curriculum planner is expecting the students to learn. The enacted object of learning is the space of learning constituted in a learning environment, i.e. what is made possible for the student to learn. The lived object of learning is the way students see, understand, and make sense of the object of learning, i.e. what students actually have learned when the teaching ends.

C. Method and Sample

As mentioned above, this study is part of a larger research programme. We have focused on two different implementations of a lab in transient response (see Chapter III below). This lab was part of a first year university level course in electric circuit theory for engineering students. Learning is to experience the world in new ways [41]. To analyse learning is to analyse new ways of enabling students to experience their world. One way to do this is to observe students' behaviour (conversation and actions) [44] during lab sessions.

We have developed a model [27, 38, 40, 45], the model of learning a complex concept, which extends Tiberghien's [4] model that distinguishes between the theory/model world and the object/event world. The different relational concepts are illustrated by "islands" (see Figure 2). Arrows show the links between the different concepts. This model may be used to analyse the intended links, or the links actually made by students, depending on whether "the intended object of learning" or "the lived object of learning" [43] is investigated. We have found the items in our model by analysing what questions the students raise during labwork [46]. Our methodology is a further development of Wickman's [47] practical epistemologies based on work by Wittgenstein [48]. See Carstensen and Bernhard [27] and Carstensen *et al.* [38], for a fuller discussion regarding the methodology.

The idea behind our model is that knowledge is built both by learning of the pieces, the islands, and by learning the whole by making explicit links. Some links seem to have to follow the whole circle, while some seem to be possible to make across. We believe that the links may become established through interaction between different pieces of knowledge. The more links that are made, the more complete the knowledge becomes (cf. Roth [8]). According to Tiberghien [49], the most difficult links to make are those that go between the two "worlds", therefore to identify those links and to explore possibilities for learning, one must identify the "problems" as well as the "potentials" (For similar results see [26]).

To study student learning *in situ*, using our model, we videotaped and transcribed students' conversations and other actions in different transient-lab sessions. In each lab session we followed two groups (comprising 2-3 students) with a video camera, and took a total of 56 h of video for study. In this paper we have included the results of this analysis, but not the transcripts (For transcripts see [27, 45]). By this method it has been possible for us to follow the dynamics of students' learning and to do an in-depth study of critical aspects for learning in this specific context.

If we had decided to use questionnaires instead of our method of videotaping and analysing students' courses of action it had not been possible for us to investigate and discover *critical aspects* for learning transient response in electric circuit theory. For example Entwistle and co-workers [50, 51] have investigated teaching-learning environments and student learning in electronic engineering using questionnaires and by interviews with selected students. Using questionnaires it is possible collect data from a large group of students, however these data are not of the same in-depth character and quality as our data. The questionnaires used by Entwistle and co-workers as well as the interviews are based on students' self-reports post-course about their learning experiences. Thus the questionnaires are based students reflected understanding, thereas our method makes its possible to investigate actual critical aspects. Based on these self-reports Entwistle and co-workers claimed that one of the main difficulties for students in learning analogue electronics were mathematical. However, from our studies of students' courses of action and analysis of critical aspects during the course we have found, as will be discussed in more detail below, that students' difficulties was not as much in the handling of mathematics as in the linking between the theory/model world and the object/event world (We have obtained similar results in an earlier work there we analysed learning of AC-electricity [26]). By our method we have obtained a more subtle understanding of for example the role of mathematics than have been possible by a questionnaire. Therefore it has been possible for us to do a fine-grained analysis of actual critical features and to reach a somewhat different conclusion than those drawn by Entwistle and co-workers.

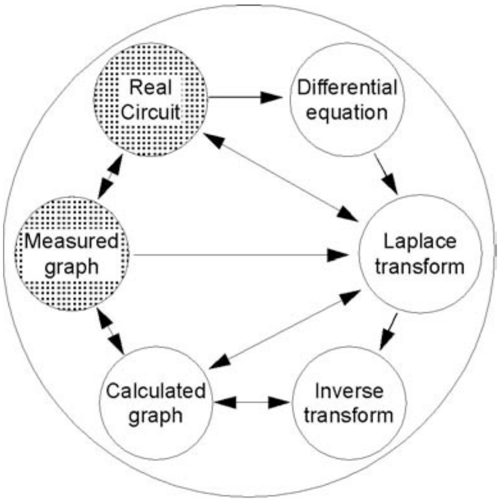


Figure 2. Our model of the learning of a complex concept used to illustrate the intended object of learning in the transient lab. The grey circles are analytically attributed to the object/event world and the other circles represent the theory/model world.

III. COURSE DESIGN AND RESULTS

A. General Design and Organisation

This project focused on a second semester course in electric circuit theory for first-year students studying electrical engineering at a Swedish university. The course included theory for DC- and AC-circuits, transient and frequency response, periodic and semi-periodic signals, and the application of transform (Phasor, Laplace and Fourier) methods and Fourier-series for solving circuit problems. Electric Circuits by Nilsson and Riedel [52] was the recommended textbook for the course at the time of our study.

Our aim was to provide experiences [35, 53] that would encourage students to relate electric circuit phenomena to their representational means (mathematical and graphical).

In the first implementations of the reformed course, in 2001 and 2002, new labs were introduced. These labs were developed by taking the experience of developing conceptual labs in physics into account [13, 23, 24, 54]; the rest of the course (lectures and problem-solving sessions) remained traditional in format and structure. Table 1 shows the general organisation of the course.

Format (2002)	# times	Length (h)	Total # hours	Nominal # students
Lecture	12	2	24	60
Problem-solving	20	2	40	30
Lab	13	2 (for 2 labs: 4h)	30	15
Total # hours for each student			94	

Table 1. The general organisation of the electric circuit theory course in 2002.

Although in our study the new labs allowed the development of a functional understanding of the concepts involved in the course, our analysis showed that further development was possible. In 2003 we redesigned the logistics of the course; the problem-solving sessions were integrated into the labs and all lab instructions were rewritten. Problems similar to those found in textbooks were integrated into the instructions. However, the 'traditional' problems were not copied from the textbooks, but were carefully reworked in light of the theory of variation presented above. Ample consideration was given to how these problems could fit into a learning environment and foster an understanding of electricity as an integrated holistic knowledge. One of the advantages of this integrated environment was that several tools were made available to the students during problem-solving: Besides paper and pencil, students had mathematical tools such as MATLAB™, toolboxes such as SIMULINK™, circuit simulation software such as PSpice, and were given the opportunity to measure real circuits. The labs were designed in such a way that the students were required to use several 'tools' to understand and handle the subject matter, including making calculations using paper and pencil, using MATLAB™ or an associated toolbox, creating simulations, making measurements on real circuits, and analysing graphs.

The organisation of the same course after the reform in 2003 is shown in Table 2. After this reform, students were only participating in lectures and in integrated problem-solving sessions. A consequence of this was that students were taught in smaller groups for a longer period. Since it was not possible to considerably increase the cost of staffing the course, the numbers of hours for a student were reduced from 94 h to 78 h. According to our analysis the higher quality of the integrated sessions more than compensated for this reduction in hours.

Format (2003)	# times	Length (h)	Total # hours	Nominal # students
Lecture	13	2	26	60
Integrated problem-solving labs	13	4	52	15
Total # hours for each student			78	

Table 2. The general organisation of the electric circuit theory course in 2003.

The following integrated problem-solving labs were developed for the reformed course in 2003:

- Voltage and current – PSpice
- Voltage and current – MATLAB™
- AC-electricity – Complex (phasor) representation
- AC-electricity – Circuit analysis
- AC-electricity – Frequency dependency
- AC-electricity – Power
- Magnetic circuits
- Transient response I
- Transient response II
- Fourier I
- Fourier II
- Mathematical methods for circuit analysis
- Summary – further problem solving.

In this paper we investigate students' behaviour in the transient response labs, labs that are carried out late in the course, before and after the 2002/2003 reform, and relate our findings to the changes we made and our theoretical framework.

B. The Intended Object of Learning – Transient Response

The intended object of learning in this lab was for students to develop a functional understanding of, and obtain some experience in, transient phenomena in electric circuits. Students were also expected to develop an ability to use different tools, such as the Laplace transform, to analyse and explain these phenomena. This is illustrated in Figure 2.

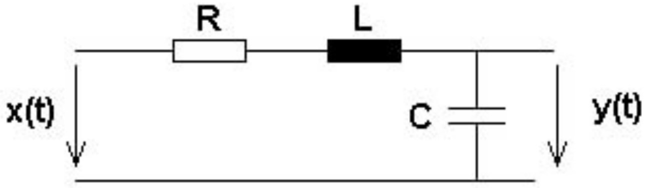


Figure 3. The circuit analysed in the transient response lab.

The circuit analysed in this lab is shown in Figure 3. For most tasks L = 8.2 mH and C = 100 µF, which was kept constant, while R was varied. The coil had a resistance of approximately 6 Ω and the external resistors (R_{Resistor}) were varied (0, 10, 33 and 100 Ω). Thus, the total resistance of the circuit was R = 6, 16, 39 and 106 Ω. The input voltage x(t) was a step-function achieved through a square wave with a long period-time and amplitude of 1 V.

The equation for the relationship between $v_{in}(t)$ and the current through the circuit can be written as:

$$x(t) = v_{in}(t) = R \cdot i(t) + L\frac{d}{dt}i(t) + \frac{1}{C}\int_{-\infty}^{t} i(t)dt$$

In our example, $v_{in}(t)$ will be given as a step-function where i(t) is sought. For most students such an integral-differential equation is difficult to solve. However, using Laplace transforms, this equation in the time-domain could be transformed to an algebraic equation in the frequency domain. Using $V_{in}(s) = 1/s$ for the voltage step and following standard procedures, it can be written as:

$$V_{in}(s) = R \cdot I(s) + sL \cdot I(s) + \frac{1}{sC} \cdot I(s) \Rightarrow$$

$$I(s) = \frac{V_{in}(s)}{R + sL + \frac{1}{sC}} = \frac{1/s}{R + sL + \frac{1}{sC}} = \frac{1}{s^2L + sR + \frac{1}{C}} = \frac{1}{L} \cdot \frac{1}{s^2 + s\frac{R}{L} + \frac{1}{LC}}$$

Depending on the relationship between R, L and C, the roots of the denominator $s^2 + sR/L + 1/(LC)$ will be complex-conjugated, a double or two real ones. Depending on the type of roots, we will get different types of functions corresponding to i(t). The types of functions corresponding to different R-values are presented in Table 3.

R (Ω)	Roots of $s^2 + s\frac{R}{L} + \frac{1}{LC}$		Function
6	-366+1042j	-366-1042j	$ae^{-366t}\sin(1042t)$
16	-976+517j	-976-517j	$be^{-976t}\sin(517t)$
39	-272	-4484	$c(e^{-272t} - e^{-4484t})$
106	-95	-12832	$d(e^{-95t} - e^{-12832t})$

Table 3. Roots of the denominator $s^2 + sR/L + 1/(LC)$ and the corresponding functions for L = 8.2 mH and C = 100 μF.

Here, it is possible to go from the *Real circuit → Differential equation → Laplace transform → Inverse transform → Calculated graph.* Finally, it is possible to compare the calculated graph with a measured graph.

It is also possible, for example, to go in the other direction: Measured graph as data points → Function fit to measured graph → Laplace transform → Real circuit.

For example, a fit to measured data in the form of:

$$i(t) = ae^{-bt}\sin\omega t \Rightarrow I(s) = a\frac{\omega}{(s+b)^2 + \omega^2}$$

can be compared to:

$$I(s) = \frac{1}{L} \cdot \frac{1}{s^2 + s\frac{R}{L} + \frac{1}{LC}}$$

in order to give R, L and C *experimentally* from the curve-fit.

Above it is mentioned that the intended object of learning in the transient response lab can be illustrated in the form of the *model for learning a complex concept,* as shown in Figure 2. It is easy to identify the links mentioned above. These links are shown for the first task in Figure 4a, and for the second task in Figure 4b. The links that associate *Real circuit → Differential equation → Laplace transform* in Figure 4b represent the links students are supposed to establish in order to make the link back: *Laplace transform → Real circuit.*

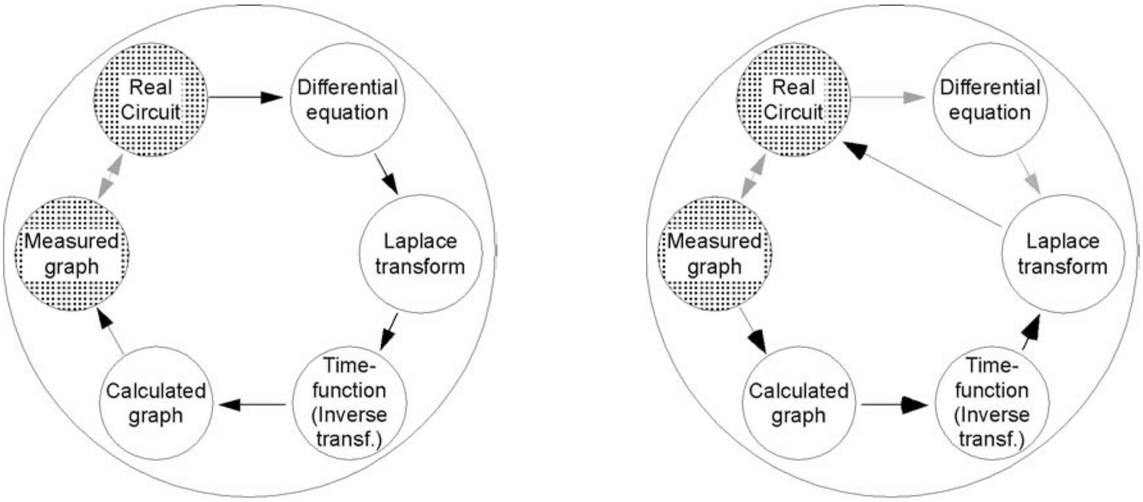


Figure 4. Analysis of different tasks (intended object of learning) in the lab-instructions (see text) in light of our model (see Figure 2) of the learning of a complex concept.

C. Design of the First Implementation and Results

In the first implementation of the transient response lab the students had one 4 h lab on transients and three 2 h sessions on problem solving.

The task was to measure the current as a function of time through the RLC-series circuit (described in the section above) for different values of R. Students were also requested to measure the voltage over the capacitor in the circuit. Figure 5 shows typical results for the RLC-circuit with the coil's own resistance (approximately 6 Ω) as the only resistance; the experimental i(t) corresponds to a damped sinus in this case.

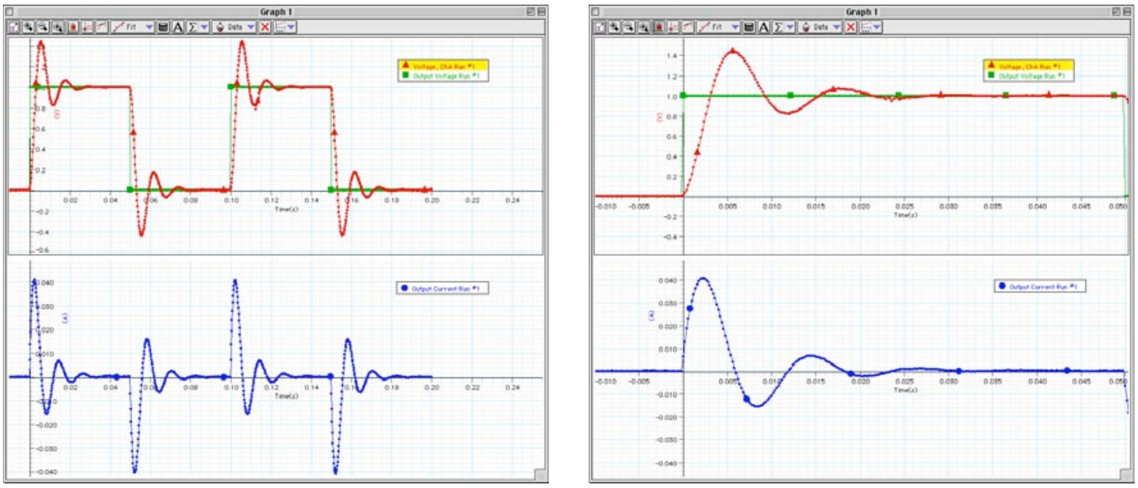


Figure 5. Results from measurement of current through the circuit (lower part) and voltage (upper part) over the whole circuit, and over the capacitor, for $R = R_{coil}$, $L = 8.2$ mH and 100 μF. The graphs to the right and left have different time scaling on the x-axis.

The experimental results for i(t) for all the different values of R are shown in Figure 6. There are two qualitatively different graphs shown that represent possible expected outcomes from that kind of input. Depending on the value of the resistor, the graph will show one or the other of the two different curves. The equations that will render the two different types of curves (see section B above) are either of type:

$$i(t) = ae^{bt}\sin(ct + d)$$

or

$$i(t) = ae^{bt} + ce^{dt}$$

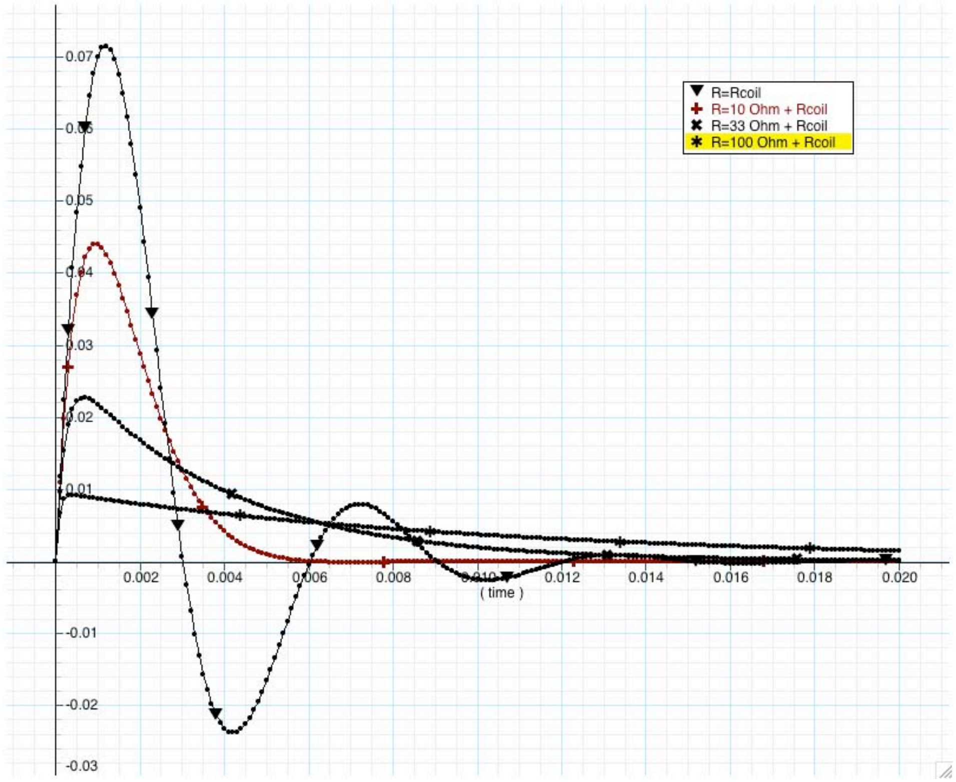
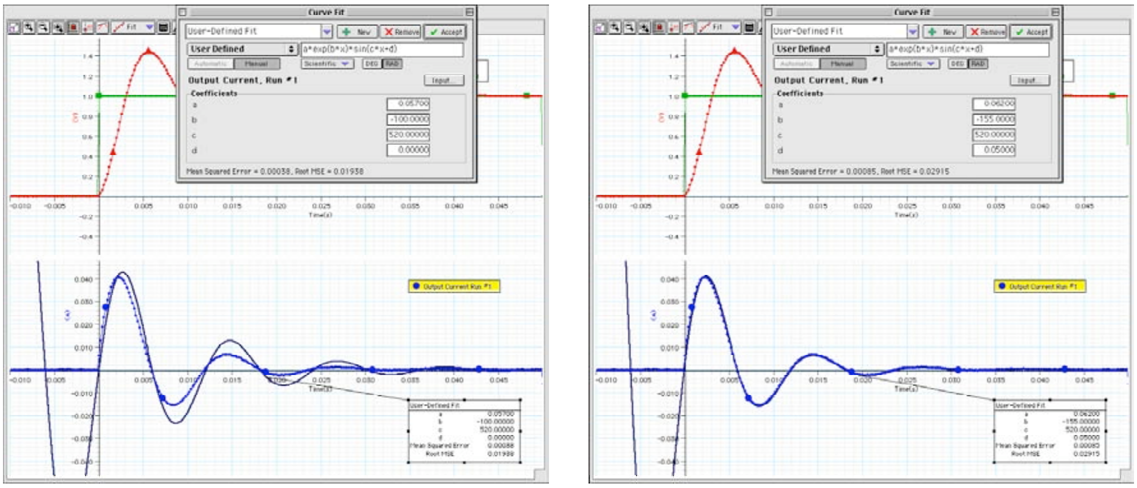


Figure 6. Typical experimental results for the current through the RLC-circuit with different resistor values.

Only one of these equations was given in the lab instruction, since one aim was to make the students aware of the different solutions to the differential equations in the context of electric circuits. This should not have proved too problematic for the students, however, who had attended previous problem-solving sessions as part of their course where both of these equations were discussed. In the MBL-environment it is possible to get both the measured and the calculated graph in the same diagram, so one task was to enter ‘the right formula’ and change the parameters a, b, c and d, until the calculated and the measured curves coincided. In Figure 7, an example is shown of how a user-defined fit is made with the software used. Since it was our intention that students should get a ‘feeling’ for what the different parameters do, students were asked to make a manual fitting and select the most appropriate function.



Curve Fit
User-Defined Fit
User Defined: a*exp(b*x)*sin(c*x+d)
Output Current, Run #1
Coefficients
a 0.05700
b -100.0000
c 520.00000
d 0.00000
Mean Squared Error = 0.00038, Root MSE = 0.01938

Curve Fit
User-Defined Fit
User Defined: a*exp(b*x)*sin(c*x+d)
Output Current, Run #1
Coefficients
a 0.06200
b -155.0000
c 520.00000
d 0.05000
Mean Squared Error = 0.00085, Root MSE = 0.02915

Figure 7. User-defined fit to experimental data. The left graph shows an unfinished fit and the right graph shows a fit that agrees well with the experimental data.

The lab instruction asked students to make curve-fits to all four different measured curves for i(t). Furthermore, they were required to calculate the corresponding R, L, C-values from the curve-fits made. We expected the students to notice and explain the differences between the different experimental curves and relate this difference to the theory.

In the design we expected that the variation in qualitatively-different types of experimental curves experienced in the lab, corresponding to qualitatively different types of roots (poles), would contribute to the students' enhanced understanding of transients and the application of Laplace transforms in electric circuit theory.

However, when we, in the first implementation of the course, analysed the students' courses of action, we noticed several 'lingering gaps' [47], for example, a question that was not answered during the lab. No students from any group noticed that they should use two different formulae for the curve-fit and tried to fit a damped sinus to the $R_{resistor}$ = 33 and 100 Ω curves. There was much surface discussion about how the curve looked instead of what it meant. The students also discussed what might be needed for the report, rather than trying to understand the curves, and relating them to things learned from lectures, textbook reading, and earlier experiments. It should be noted that no student used notes from lectures, classroom sessions or the textbook without being explicitly told to do so. Our results show that it was still very difficult for students to make links between the theory/model world and the object/event world, despite the aim of the design to deliberately facilitate this linkage.

D. Design of the Second Implementation and Results

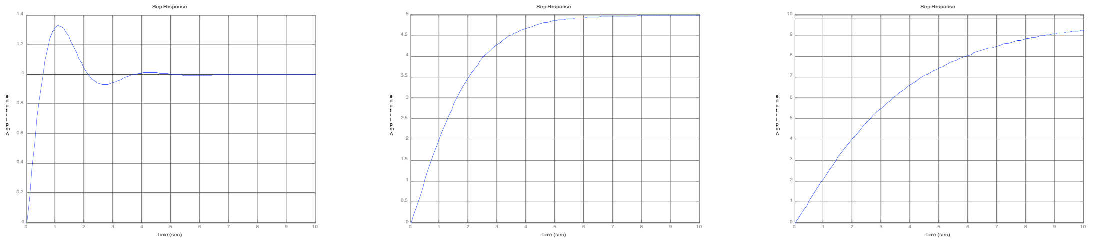
The aim of integrating problem-solving sessions and labs was to further widen the students' opportunity to experience the links between the world of object and events and the world of theory and models. In the first implementation, the transient response lab lasted for four hours and the classroom sessions devoted to solving transient response and Laplace problems comprised of three two-hour sessions; a total of 10 hours. In the second implementation this was transformed in to two four-hour sessions; a total of 8 hours.

The main difference in the second implementation of the transient lab was that the lab started with the six different Laplace transforms displayed in Figure 8. Students were first asked to solve the problems using paper and pencil, then to simulate them with SIMULINK™.

a) $G(s) = \dfrac{2s+5}{s^2+2s+5}$

b) $G(s) = \dfrac{2s+5}{s^2+2s+1}$

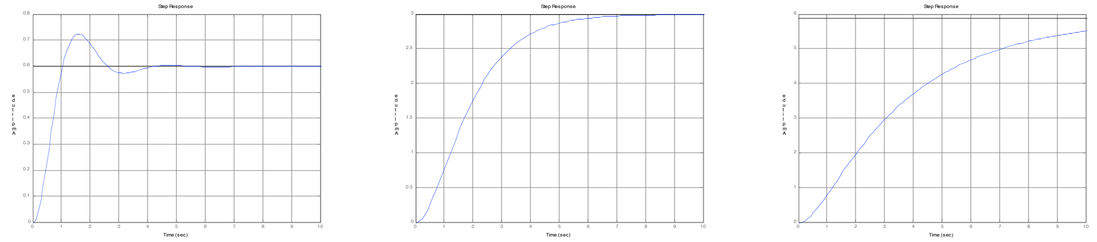
c) $G(s) = \dfrac{2s+5}{s^2+2s+0.51}$



d) $G(s) = \dfrac{3}{s^2+2s+5}$

e) $G(s) = \dfrac{3}{s^2+2s+1}$

f) $G(s) = \dfrac{3}{s^2+2s+0.51}$



Important characteristics:

1) Solutions to the characteristic polynomial, i.e. the poles to the transfer function give different shapes to the curves:

$$s = -1 \pm \sqrt{1-5}$$
$$s_1 = -1 + 2j$$
$$s_2 = -1 - 2j$$

gives under-critically damped behavior

$$s = -1 \pm \sqrt{1-1}$$
$$s_{1,2} = -1$$

gives critically damped behavior

$$s = -1 \pm \sqrt{1-0.51}$$
$$s_1 = -1 + 0.7 = -0.3$$
$$s_2 = -1 - 0.7 = -1.7$$

gives over-critically damped behavior

2) Note the different start behaviour that depends on the difference in degree of powers in the numerator and denominator polynomials.

3) The Steady-State value depends on the transfer-function's limit-value when s approaches zero.

Final value theorem $\lim\limits_{t \to \infty} g(t) = \lim\limits_{s \to 0} G(s)$

Figure 8. Examples of systematically-varied Laplace functions to analyse, mathematically and graphically, in the transient lab.

In Figure 8, the three different denominator polynomials correspond to three qualitatively different types of solutions. The resulting time functions g(t) are also shown above. Besides experiencing the variation, due to the different characteristics of the roots to the characteristic polynomia, i.e. different types of poles, variation is also experienced according to the initial and final value theorems, as explained in point 2 and 3.

Typical 'end of chapter' conclusions in most textbooks are not systematically varied, as seen in Figure 8. In the first implementation these traditional textbook problems were used. By using SIMULINK™ the characteristics of the different functions were visualised.

The next step in the instructions required students to work out the relationship G(s) = Y(s)/X(s) = $V_C(s)/V_{in}(s)$ with paper and pencil from Figure 3. Students were then asked to inverse transform this relationship and calculate $V_C(t) = y(t)$ for some of the values of R, L and C that occur in the RLC-circuit.

After this step students were asked to begin taking measurements of the transient response of the real circuit. The task, the intended object of learning, in this part of the lab was very similar to those in the first version of the transient lab. However, the likeness in instruction in this part of the lab, the students' course of action, and the lived object of learning, were very different in the two versions of the lab.

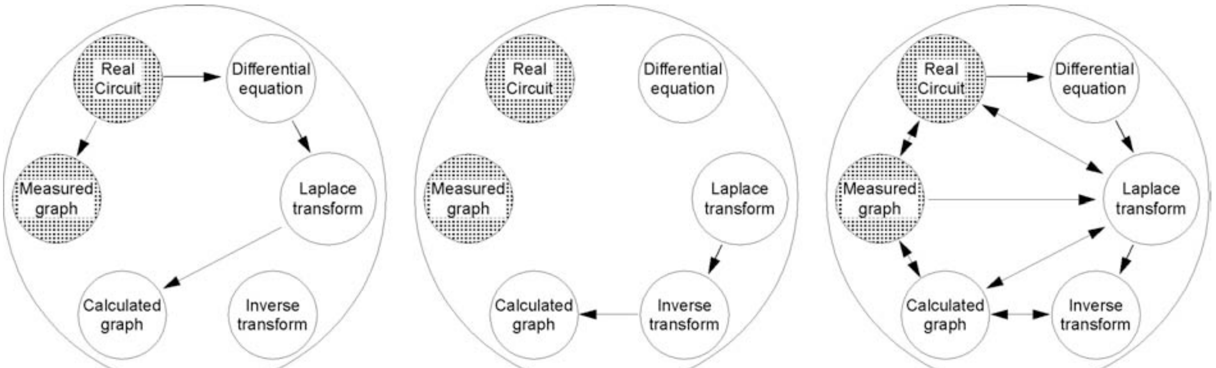


Figure 9. An example of an analysis of learning in the transient lab, using the model for learning a complex concept: a) Student Benny's lived object of learning after the first part of the revised lab: b) Student Tess' lived object of learning at the same time; and c) Student Benny's and Tess' lived object of learning at the end of the lab work in the revised lab.

To demonstrate how our analysis of learning was made, please refer to Figure 9a-c. After the first part of the lab, the lived object of learning for the two students Benny and Tess (respectively), could be described with the arrows displayed in figure. In making our analysis we studied transcripts and video tapes from the lab. It is clear that Benny and Tess were making different links and not communicating clearly. In the transcripts taken from the later parts of the lab, Benny and Tess eventually engaged. From the analysis, it is also obvious that the students had difficulties connecting the mathematical representation to the measured graphs and the circuit they used. Tess and Benny encountered different objects of learning, and in order to fill the gaps they had to make links back to what they already knew. At this point, neither student was thinking about the real circuit, because in order to do so they had to make links back – Benny from the graph and Tess from the mathematics. At the end of the lab session Tess and Benny had made all the links described in Figure 9c. Their discussion simultaneously covered two or more of the links, and their awareness of the other links were figurative, so that they drew their conclusions from what they saw.

E. Comparison and Analysis

Common to the two implementations described above, were the measurement, and the modelling of the step-response of the current through the RLC-circuit. In the first implementation, this measurement and modelling were the students' main task. In this task we had variance in the value of R and invariance in the L and C-values and in the circuit topology. The variance in R led to variance in characteristics of the step-response. From our analysis we found that, although it was our intended object of learning, in the first implementation of the transient lab students did not establish the links displayed in Figure 2. Students' linked object of learning did not correspond to the intended object of learning. Students did not, in the first implementation, discern all the critical aspects.

In the second implementation of the transient lab the task described in Figure 8 preceded the measurements on the RLC-circuit. Our analysis showed that the variance introduced in this task was vital for students to be able to identify the critical aspects of the object of learning. Marton et al. [1, p. 21] point out "[v]ariation that is not present in the situation can still be discerned … if variation is brought in by means of … previous experience". They also point out that (p. 30) "it is very important that the teacher is able to bring critical features of the object of learning into students' focal awareness". By introducing the tasks before the measurements, in the design of the second implementation, critical features of the object of learning were introduced to the students' awareness.

A common question in the first implementation was: Is this curve good enough for the report? This question was not asked in the second implementation. We believe this question creates a lingering gap [47], i.e. a question that is not answered during the lab. The students sometimes received hints from the teacher on how to proceed, e.g. the teacher asked the students to refer back to their notes and do some calculations. It is interesting that the students did not, even when asked by the teacher to do so, open their notes during the lab. The students felt it was a waste of time to do theory in the laboratory, although the teacher told them that it was the only way to get an idea of which curves it was possible to expect.

In the new course, the students knew that there was time for both calculations and lab-work, and they demonstrated this by working differently. The trial-and-error behaviour seen in the old course disappeared. At the beginning of this session the discussions within and between groups concerned the subject matter. They very soon found patterns which made it possible for them to compare the calculated graphs and the measured graphs [27].

IV. CONCLUSIONS AND IMPLICATIONS

The conclusion of our work has several important implications for future designs. Firstly, integrating the lab sessions and the problem-solving sessions gives students new ways to handle the subject matter. They bring their knowledge from the mathematical context into the lab, but can also use the graphs when elaborating the mathematical context. When simultaneously working from the object/event world, as well as the theory/model worlds, the students make the vital link. From this, the focus of the lab work is changed. Instead of focusing on what to report, the students now focus on what is to be learned, i.e. they make links between all the components of the circle in our model.

Secondly, the study shows the importance of conducting a fine-grained analysis of students' courses of action in education. Without this careful analysis, we would not have seen that our intended object of learning was not the enacted object of learning in the first implementation of the lab. Our results show that our model of learning of a complex concept, and Marton's theory of variation, are valuable tools to analyse the lived object of learning. Our results also indicate that using the theory of variation is useful in the design and improvement of learning environments. Existing learning environments could be analysed by looking at what kind of variation is afforded by the design, if any. These findings could then be taken into account, as we have done, by re-designing the learning environment by introducing the necessary variation missing in the original design. Thus, the enacted object of learning is improved, leading to an improved lived object of learning.

Thirdly, our results reconfirm that the links between the object/event world and the theory/model world have to be made explicit in lab work.

Finally, our research shows that Laplace transforms are not too difficult to teach and learn, and that it is possible to achieve a functional understanding of transient response.

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REFERENCES

[1] Marton, F., U. Runesson, and A.B.M. Tsui, "The Space of Learning", *Classroom Discourse and the Space of learning*, Mahwah: Lawrence Erlbaum, 2004, pp. 3-40.

[2] Rorty, R., *Objectivity, Relativism, and Truth: Philosophical Papers Volume 1*, Cambridge: Cambridge University Press, 1991.

[3] Carstensen, A.-K., and J. Bernhard, "Critical aspects for learning – making links between the object/event world and the theory/model world explicit in labwork", *SEFI 2006*, Uppsala, 2006.

[4] Tiberghien, A., "Labwork activity and learning physics - an approach based on modelling", *Practical Work in Science Education*, Dordrecht, The Netherlands: Kluwer, 1999, pp. 176-194.

[5] Vince, J., and A. Tiberghien, "Modelling in Teaching and Learning Elementary Physics", *The Role of Communication in Learning to Model*, Mahwah: Lawrence Erlbaum, 2002, pp. 49-68.

[6] McDermott, L.C., "How Research Can Guide Us In Improving the Introductory Course", *Conference on the Introductory Physics Course: On the Occasion of the Retirement of Robert Resnick*, New York: John Wiley & Sons, 1997, pp. 33-46.

[7] Thornton, R.K., "Learning Physics Concepts in the Introductory Course: Microcomputer-based Labs and Interactive Lecture Demonstrations", *Proceedings Conference on Introductory Physics Course*, New York: Wiley, 1997, pp. 69-86.

[8] Roth, W.-M., *Authentic school science: Knowing and learning in open-inquiry science laboratories*, Dordrecht: Kluwer, 1995.

[9] Roth, W.-M., and M.G. Bowen, "Professionals Read Graphs: A Semiotic Analysis", *Journal for Research in Mathematics Education* Vol. 32, 2001, pp. 159-194.

[10] Psillos, D., Niedderer, H., *Teaching and learning in the science laboratory*: Kluwer Academic Publishers, 2002.

[11] Thornton, R.K., and D.R. Sokoloff, "Assessing student learning of Newton's laws: The Force and Motion Conceptual Evaluation and the Evaluation of Active Learning Laboratory and Lecture Curricula", *American Journal of Physics* Vol. 66, No. 4, 1998, pp. 338-352.

[12] Tinker, R.F., "Introduction", *Microcomputer-Based Labs: Educational Research and Standards*, Berlin: Springer, 1996, pp. 1-10.

[13] Bernhard, J., "Experientially based Physics Instruction using hands on Experiments and Computers." *PTEE 1997*, Copenhagen, 1997.

[14] Bernhard, J., "Experientially based physics instruction - using hands on experiments and computers: Final report of project 167/96", Stockholm: Council for Renewal of Higher Education, 2005.

[15] Bernhard, J., "Developing and studying conceptual labs - experiences from a decennium of design and implementation", *ESERA Summerschool*, Braga, 2006.

[16] Bernhard, J., "Experientially based physics instruction: A symbiosis between American and European thinking", *AAPT Summer Meeting*, Greensboro, 2007.

[17] Cobb, P., J. Confrey, A. diSessa, R. Lehrer, and L. Schauble, "Design experiments in educational research", *Educational Researcher* Vol. 32, No. 1, 2003, pp. 9-13.

[18] Barab, S., and K. Squire, "Design-based research: Putting a stake in the ground", *Journal of the Learning Sciences* Vol. 13, 2004, pp. 1-14.

[19] Fishman, B., R. Marx, P. Blumenfeld, J. Krajcik, and E. Soloway, "Creating a framework for research on systemic technology innovations", *Journal of the Learning Sciences* Vol. 13, 2004, pp. 43-76.

[20] Design-Based Research Collective, "Design-based research: An emerging paradigm for educational inquiry", *Educational Researcher* Vol. 32, No. 1, 2003, pp. 5-8.

[21] Runesson, U., *Variationens pedagogik. Skilda sätt att behandla ett matematiskt innehåll.*, Göteborg: Göteborg Studies in Educational Sciences, 1999.

[22] Lo, M.L., F. Marton, M.F. Pang, and W.Y. Pong, "Toward a Pedagogy of Learning", *Classroom Discourse and the Space of learning*, Mahwah: Lawrence Erlbaum, 2004, pp. 189-225.

[23] Bernhard, J., "Teaching engineering mechanics courses using active engagement methods", *PTEE 2000*, Budapest, 2000.

[24] Bernhard, J., "Physics learning and microcomputer based laboratory (MBL): Learning effects of using MBL as a technological and as a cognitive tool", *Science Education Research in the Knowledge Based Society*, Dordrecht: Kluwer, 2003, pp. 313-321.

[25] Bernhard, J., and A.-K. Carstensen, "Designing conceptual labs", *PTEE 2005*, Brno, 2005.

[26] Bernhard, J., and A.-K. Carstensen, "Learning and teaching electrical circuit theory", *PTEE 2002: Physics Teaching in Engineering Education*, Leuven, 2002.

[27] Carstensen, A.-K., and J. Bernhard, "Laplace transforms - too difficult to teach learn and apply, or just matter of how to do it." *EARLI sig#9 Conference*, Göteborg, 2004.

[28] Vygotsky, L.S., *Mind in society: The development of higher psychological processes*, Cambridge: Harvard University Press, 1978.

[29] Davydov, V.V., and S.T. Kerr, "The Influence of L. S. Vygotsky on Education Theory, Research, and Practice", *Educational Researcher* Vol. 24, No. 3, 1995, pp. 12-21.

[30] Kozulin, A., B. Gindis, V.S. Ageyev, and S.M. Miller, (Eds), Vygotsky's Educational Theory in Cultural Context, Cambridge: Cambridge University Press, 2003.

[31] Cole, M., *Cultural psychology: A once and future discipline*, Cambridge: Harvard University Press, 1996.

[32] Wertsch, J.V., *Mind as Action*, Oxford: Oxford University Press, 1998.

[33] Dewey, J., "Logic: The Theory of Inquiry", *John Dewey: The Later Works* (Vol. 12), Carbondale: Southern Illinios University Press, 1986 (original work published 1938).

[34] Dewey, J., "Logical Objects", *John Dewey: The Middle Works* (Vol. 10), Carbondale: Southern Illinois University Press, 1980 (original work published 1916), pp. 89-97.

[35] Dewey, J., "Experience and Nature", *John Dewey: The Later Works* (Vol. 1), Carbondale: Southern Illinois University Press, 1981 (original work published 1925).

[36] Kress, G., C. Jewitt, J. Ogborn, and C. Tsatsarelis, *Multimodal Teaching and Learning: The Rhetorics of the Science Classroom*, London: Continuum, 2001.

[37] Marton, F., and A.B.M. Tsui, (Eds), Classroom Discourse and the Space of Learning, Mahwaw: Lawrence Erlbaum, 2004.

[38] Carstensen, A.-K., M. Degerman, and J. Bernhard, "A theoretical approach to the learning of complex concepts", *ESERA 2005*, Barcelona, 2005.

[39] Bernhard, J., and A.-K. Carstensen, "Modelling and learning a complex concept - an exploration in light of some examples from electric circuit theory", *ESERA 2007*, Malmö, 2007.

[40] Bernhard, J., A.-K. Carstensen, and M. Holmberg, "Analytical Tools in Engineering Education Research - the Model of a Complex Concept, Threshold Concepts and Key Concepts in Understanding of and Designing for Student Learning", 2007.

[41] Marton, F., and S. Booth, *Learning and Awareness*, Mahwah: Erlbaum, 1997.

[42] Bowden, J., and F. Marton, *The University of Learning: Beyond Quality and Competence in Higher Education*, London: Kogan Page, 1998.

[43] Runesson, U., and F. Marton, "The Object of Learning and the Space Variation", *What Matters? Discovering critical conditions of classroom learning*, Göteborg: Acta Universitatis Gothoburgensis, 2002, pp. 19-38.

[44] Jordan, B., and A. Henderson, "Interaction Analysis: Foundations and Practice", *The Journal of the Learning Sciences* Vol. 4, No. 1, 1995, pp. 39-103.

[45] Carstensen, A.-K., *Connect: Models and the Real World Connected Through Lab-Work in Electric Circuit Theory*, Linköping: Linköping Studies in Science and Technology Dissertation, 2007.

[46] Carstensen, A.-K., and J. Bernhard, "What questions are raised during lab-work?" *ESERA2003*, Nordwijkerhout, 2003.

[47] Wickman, P.-O., "The Practical Epistemologies of the Classroom: A Study of Laboratory Work", *Science Education* Vol. 88, 2004, pp. 325–344.

[48] Wittgenstein, L., *Philosophische Untersuchungen*, Frankfurt am Main: Suhrkamp Verlag, 2003 (original work published 1953).

[49] Tiberghien, A., "Designing teaching situations in the secondary school", *Improving science education*, Buckingham: Open University Press, 2000, pp. 27-47.

[50] Entwistle, N.J., J. Nisbet, and A. Bromage, "Teaching-learning environments and student learning in electronic engineering", *Third Workshop of the European Network on Powerful Learning Environments*, Brugge, 2004.

[51] Entwistle, N.J., A. Hamilton, R.G. Kelley, J.B. Nisbet, R. Chapman, G. Hayward, and A. Gachagan, "Teaching and learning analogue electronics in undergraduate courses: preliminary findings from the ETL project", *International Journal of Electrical Engineering Education* Vol. 42, No. 1, 2005, pp. 8-20.

[52] Nilsson, J.W., and S.A. Riedel, *Electric Circuits*, Sixth ed., Upper Saddle River: Prentice-Hall, 2001.

[53] Dewey, J., *Experience & Education*, New York: Touchstone, 1938.

[54] Bernhard, J., "Hands-on experiments in advanced mechanics courses", *Hands on-Experiments in Physics Education*, Duisburg: Didaktik der Physik, University of Duisburg, 1999, pp. 175-177.