

Paper presented at the 1997 Summer Seminar *Teaching Introductory Physics Using Interactive Teaching Methods and Computers*.

Dickinson College, Carlisle, PA, USA, June 15 – 27, 1997.

The coupled harmonic oscillator

– A lab using MBL-tools for an upper-level mechanics course

Jonte Bernhard
Sektionen för Naturvetenskap och Teknik
Högskolan Dalarna
S-781 88 Borlänge, Sweden
E-mail: jbe@du.se
Homepage: <http://www.du.se/~jbe> (under construction)

Introduction and background

At Högskolan Dalarna we made major changes to our introductory mechanics for engineering students last year. The course is now more conceptual and gives a broader overview of mechanics but on a lower mathematical level than before. The mechanics lab-sequence was remodelled using MBL-equipment and software like Videopoint, Graphs and Tracks and Interactive Physics. MBL-equipment were also used for some of the lecture demonstrations. Although the introduction of labs reduced the number of lecture/recitation hours the post-test showed that the introductory mechanics students (first year) performed better on the FCI-test, than the second year students did who had taken the advanced (old curricula) mechanics course who have had three times as many lecture/recitation hours.

This coming academic year the new curricula advanced mechanics will be given for the first time in the second quarter. This course will be on a higher mathematical level and probably we will use Fowles & Cassidy: *Analytical Mechanics* or Marion & Thornton: *Classical Dynamics*. The aim of my part of the project were to look for good labs which could be included in an upper-level mechanics course. Below I will discuss one lab, using MBL-equipment, which could be implemented in an upper-level mechanics course.

Coupled Harmonic Oscillator

One topic typically discussed in an upper-level mechanics course is the coupled harmonic oscillator. The concept of different normal vibrational modes introduced with a coupled oscillator is of great importance in science and engineering.

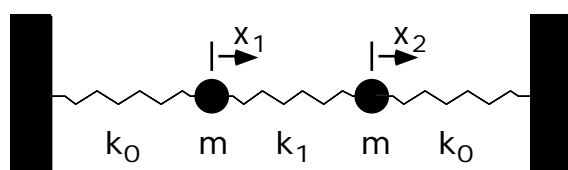


Fig 1. Model of a simple coupled harmonic oscillator.

The equations for a coupled harmonic with two equal masses can be written as

$$m\ddot{x}_1 + k_0x_1 - k_1(x_2 - x_1) = 0$$

$$m\ddot{x}_2 + k_0x_2 + k_1(x_2 - x_1) = 0$$

Unfortunately this differential equation is nontrivial to solve due to the coupling term. Solving this differential equation we get two so called vibrational "normal" modes, corresponding to the masses oscillating in either phase (symmetric mode) or antiphase (antisymmetric mode). All other oscillations of the masses can be described as linear combinations (superposition) of the "normal" modes. Thus the general solution for one of the masses can be written as

$$x_1(t) = A_a \cos(\omega_a t + \phi_a) + A_s \cos(\omega_s t + \phi_s)$$

there the subscripts a and s refer to the antisymmetric and symmetric modes respectively. The angular frequencies are determined by the masses and spring constants involved and the amplitudes and phases are set by initial conditions.

However most student do not get any conceptual understanding of the meaning of vibrational modes or the superposition involved in an arbitrary oscillation from only solving the differential equations (or probably only watching the professor demonstrate the solution on the blackboard!).

Fortunately MBL-equipment allows us to study the coupled harmonic oscillator experimentally. The experimental set-up is displayed in figure 2 below. PASCO physical pendulum discs were attached to two PASCO rotary motion sensors. To a first approximation this system can be treated as a coupled harmonic oscillator. The rotary motion sensors were connected to a PASCO Science Workshop 700 interface allowing angular positions of both discs to be monitored simultaneously in real-time. The results are shown in figure 3 below.

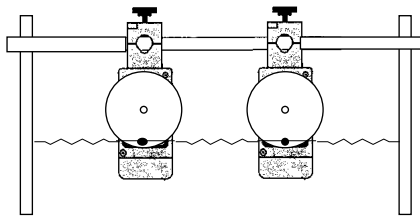


Fig 2. Experimental set-up. Two physical pendulums are mounted on PASO rotary motion sensors.

In figure 3 the result of exciting the oscillator into different oscillation modes with two different spring constants are displayed as appearing on the computer screen using PASCO Science Workshop on a Macintosh. The symmetric mode oscillation is achieved by moving both discs equal angle and in the same direction from the equilibrium position. The antisymmetric mode oscillation is achieved by moving the discs in opposite directions. A mixed mode is achieved by setting the system to oscillate without any constraints. The Frequency Spectrum graph in figure 3 displays an fast fourier transform (FFT) analysis of the oscillations using the FFT-tool in PASCO Science Workshop software. Note that the phase and antiphase behaviour of the symmetric and antisymmetric normal modes can clearly be seen. Also note that normal mode oscillations displayed are not completely pure: Small traces of oscillations of the other mode can be seen in the Frequency Spectrum. Since the measurements have to be started before the oscillator is set to swing, to obtain a zero equilibrium angular position value, a zero component can be seen in the Frequency Spectrum.

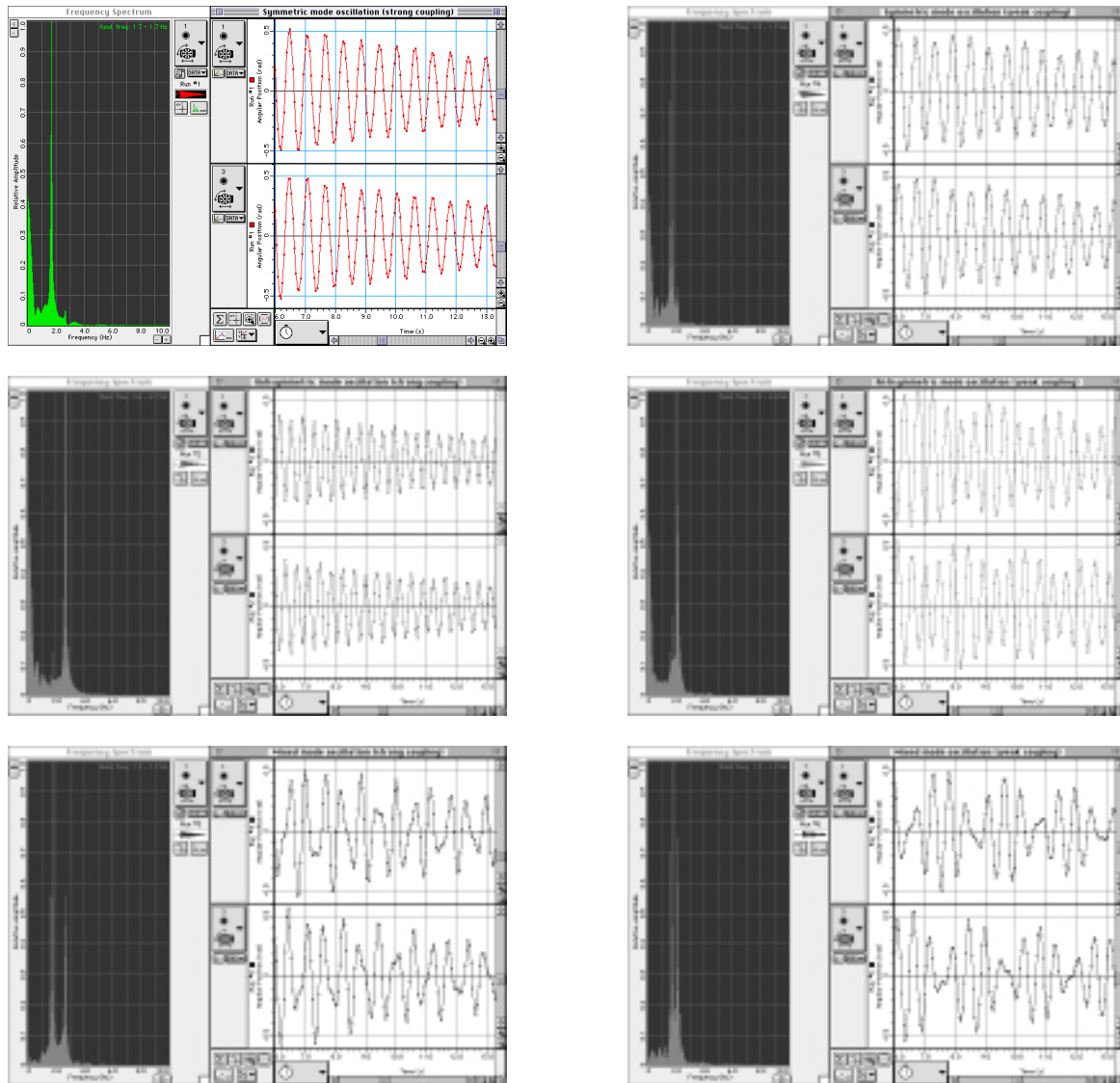


Fig 3. Results of measurements of oscillations in different modes as seen on computer screen using PASCO Science Workshop interface and software. To the left are shown oscillations using a coupling spring with larger k -value (stronger coupling) than to the right. The Angular Position vs time graph shows the displacement from the equilibrium position for both physical pendulum discs. The Frequency Spectrum graph shows the results of an FFT-analysis for the Angular Position vs time data obtained for the rotary motion sensor connected to digital channel 1 on the interface.

The experimental results and FFT-routine in Science Workshop show, that a mixed mode oscillation who on a displacement vs. time graph looks very complicated, can be resolved into two superposed normal mode oscillations.

However the FFT-routine work like a "black box" for the students if they have not been introduced to this tool in a numerical analysis course or by some frequency analysis lab. The FFT-routine takes any data showing some periodic behaviour and resolves them into which frequencies should be used to describe the data as a sum of sines or cosines.

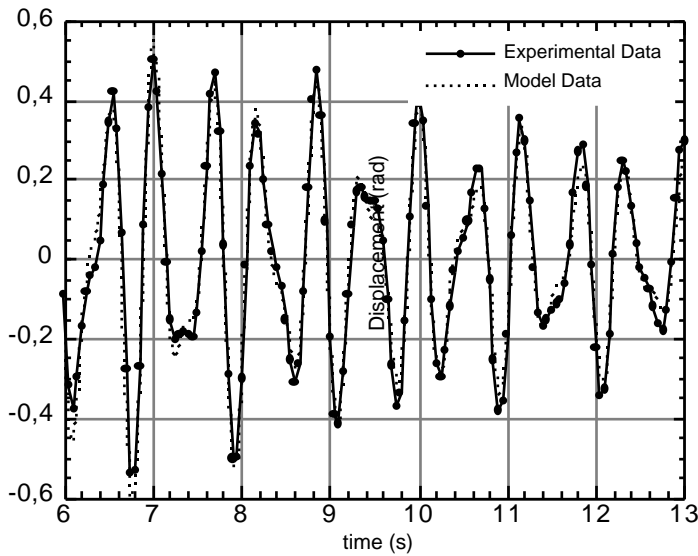


Fig 4. Experimental data for the mixed mode oscillation (strong coupling) compared to a model fit using superposition of two normal modes with damping (see text).

By using a spreadsheet it can easily be verified that the normal mode frequencies determined by FFT can be used to construct the signal. In theory once the normal modes frequencies are determined one "only" have to determine the amplitudes and the phases. The situation is however complicated because damping is involved also be the fact that that the fit to experimental data is very sensitive to small errors in normal mode frequencies. Thus if one tries to fit to experimental mixed mode data including damping one have 7

independent parameters in the model equation! However by starting fitting model data to symmetric and antisymmetric experimental data (setting the other components amplitude to zero) both normal mode frequencies and the damping term are easily determined. The normal mode frequencies obtained from FFT are used as initial values in the manual fit. When trying to fit the model to a mixed mode oscillation these values are used as fixed values and one *only* have to adjust the amplitudes and phases to obtain a good fit. Also the Frequency Spectrum of the mixed mode can be used to determine a first guess of the relative amplitudes of the two different normal modes. Going through these steps one obtain

$$x_1(t) = e^{-0,13t} [0,98 \cos(2 \ 1,691 t + 3,85) + 0,55 \cos(2 \ 2,646 t + 5,85)]$$

as being a good fit to the mixed mode strong coupling data shown in figure 3. A comparison between measured and the "model" data according the equation above is shown in figure 4.

Alternative approaches and set-ups

The PASCO Science Workshop 700 interface have been used since it could monitor two rotary motion sensors simultaneously. If Vernier ULI or PASCO Science Workshop 500 interface is used only one sensor can be monitored at the same time. Although I consider that it is important to be able to monitor both sensors simultaneously to see the difference between the symmetric and antisymmetric normal modes, the concept of superposition and FFT-analysis cab readily be approached by the other interfaces. The harmonic oscillator can also set-up using springs and gliders

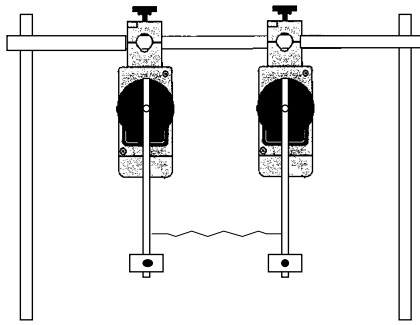


Fig 5. Coupled Harmonic oscillators using pendulums.

pendulums can be monitored. Since only one pendulum rod was available and the did not permit manufacturing one more, this set-up could not be tested while at Dickinson College. I will test it at my institute during this summer.

Discussion

The experiment described in this paper is one "advanced" experiment made feasible by the MBL-tools. Time does not permit the presentation of an actual activity guide, but I believe that is it important that the students excites several mixed mode oscillations to "verify" that all coupled oscillations can be treated as superpositions of the normal mode oscillations. I have ideas for other "advanced" experiments. Neither time nor space permit my discuss them in this paper. Since we will give an advanced mechanics second quarter this fall semester some of the ideas will be tested on real students by christmas. Any one interested should feel free to contact me.

on an air-track¹ or PASCO carts on a PASCO-track and measure the motion of one of the gliders/carts with an motion detector. In this case the motion of only one glider/cart can be measured. However, using the Vscope interface the motion of several gliders/carts can be monitored². An alternative set-up using two "ideal" pendulums coupled together with a spring are shown in Fig 5. Using the PASCO interface the motion of both

¹ Such a set-up is described in the 1992 Summer seminar proceedings and in N Preyer: *The Coupled Harmonic Oscillator, Not Just for Seniors Anymore* The Physics Teacher **34** (1996) 52–55.

² This have been done by the author at Högskolan Dalarna. However, the Vscope interface is quite expensive (3000 USD) which would be to costly for many institutions. With Vscope 3-dimensional motion for several objects can be monitored, but it does not support other sensors making it less versatile. One could settle for the 1-dimensional version which is about half price.