Critical aspects for learning in an electric circuit theory course

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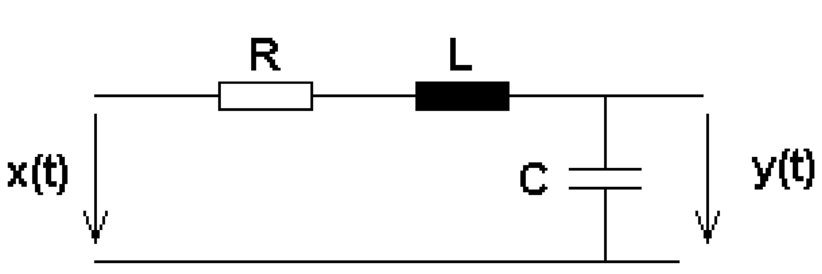
Alternating currents and transient response are considered as one of the more difficult parts of learning electric circuit theory. What makes it difficult is that the mathematics involved are rather advanced, complex representations (phasors) are used and the Laplace transform is applied to solve differential equations.

We have developed a course in electric circuit theory for engineering students, in which we have applied our earlier experiences from developing conceptual labs for mechanics and the theories of "design-based research" and the "theory of variation".

A central tenet of variation theory (developed by Ference Marton and co-workers) is that *we learn through the experience of difference*, rather than the recognition of similarity. According to this theory, *discernment*, *simultaneity* and experience of *variation* are necessary for learning.

In this course we also merged problem-solving sessions and lab-sessions, allowing the students to work in an integrated, "authentic" way on solving tasks using tools such as paper and pencil, MATLAB®, simulations and experiments. The theory of variation was applied in task design and students' courses of action were recorded during labs by by digital camcorders and subsequently analyzed.

We will briefly describ a task in which the students investigated transient responses of the current through an RLC-circuit using transform methods.



$$x(t) = v_{in}(t) = R \cdot i(t) + L\frac{d}{dt}i(t) + \frac{1}{C}\int_{-\infty}^{t} i(t)dt$$

Using standard methods the Laplace-transform for the transient response (1 V step) of the current is obtained:

$$I(s) = \frac{V_{in}(s)}{R + sL + \frac{1}{sC}} = \frac{1/s}{R + sL + \frac{1}{sC}} = \frac{1}{L}\cdot\frac{1}{s^2 + s\frac{R}{L} + \frac{1}{LC}}$$

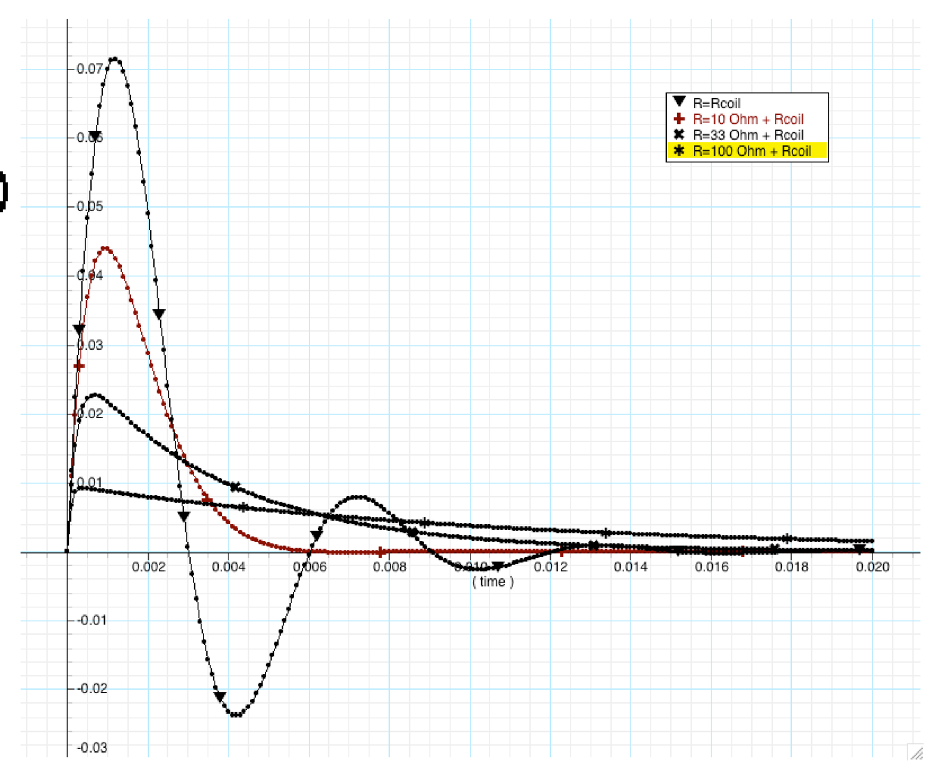
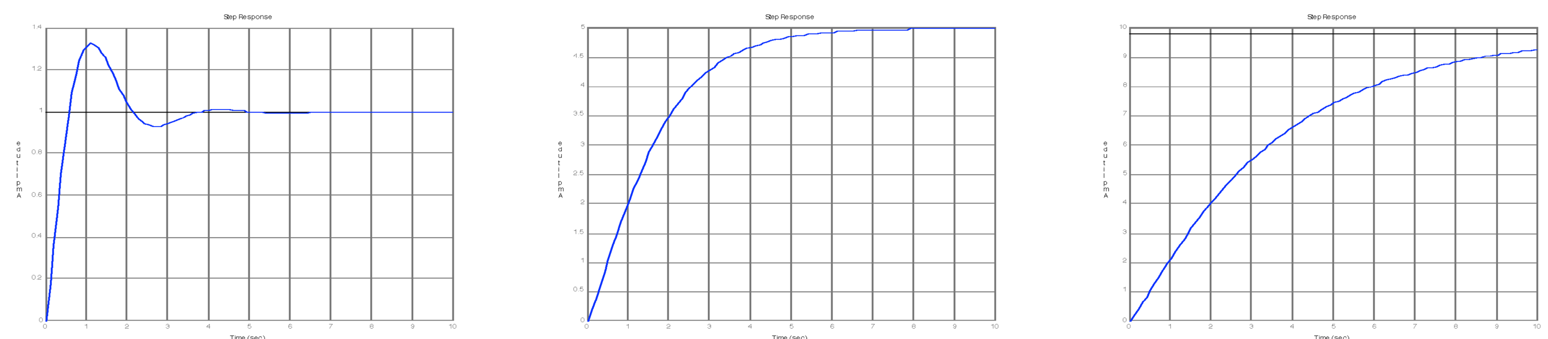


Figure 1. The circuit analysed in the transient response lab with the integral-differential equation and the Laplace-transform representing the current through the circuit. Typical experimental results for i(t) is also depicted (L = 8.2 mH and C = 100 μF).

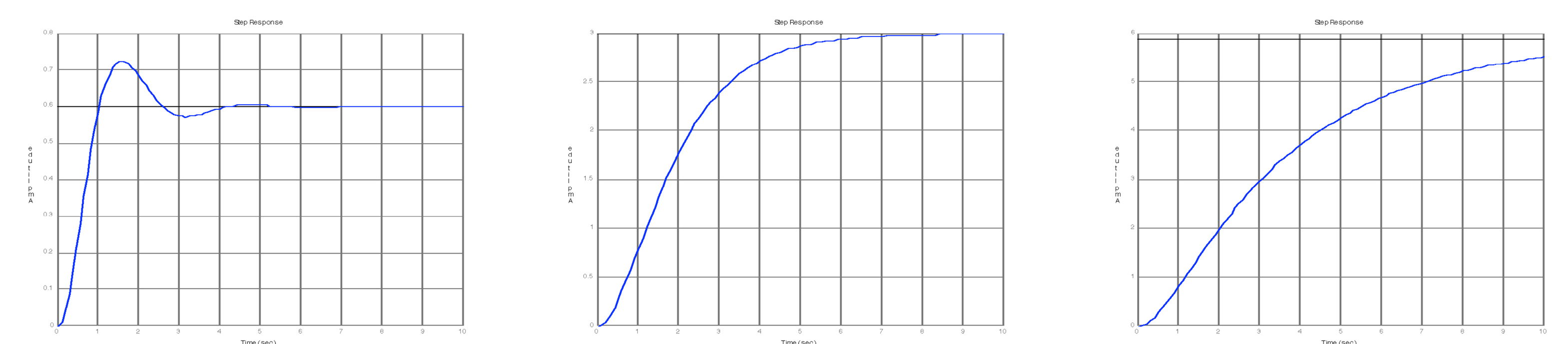
Depending on the R, L and C-values, the roots of the denominator, $s^2+sR/L+1/(LC)$, of the Laplace-transform will be complex-conjugated, a double or two real roots. These roots (poles of the transfer function) correspond to different types of functions for i(t). In a complex conjugated case ($a \pm j\omega$) we will obtain a function of type $ke^{at}\sin(\omega t+\phi)$ and with two real roots (b, c), $k_1e^{bt}+k_2e^{ct}$ (in most real cases a, b and c are negative).

In most of the lab tasks we have varied R, and kept L and C constant. In one task the students are asked to determine Z(s), i.e. R, L and C, from the experimental curves illustrated in figure 1. To do so students must be able to discern the different types of functions. The understanding of the relationship between measured and calculated graphs is also essential.

a) $G(s) = \frac{2s+5}{s^2+2s+5}$ b) $G(s) = \frac{2s+5}{s^2+2s+1}$ c) $G(s) = \frac{2s+5}{s^2+2s+0.51}$



d) $G(s) = \frac{3}{s^2+2s+5}$ e) $G(s) = \frac{3}{s^2+2s+1}$ f) $G(s) = \frac{3}{s^2+2s+0.51}$



Important characteristics:
1) Solutions to the characteristic polynomial, i.e. the poles to the transfer function give different shapes to the curves:

$s = -1 \pm \sqrt{1-5}$ $s_1 = -1 + 2j$ $s_2 = -1 - 2j$	$s = -1 \pm \sqrt{1-1}$ $s_{1,2} = -1$	$s = -1 \pm \sqrt{1-0.51}$ $s_1 = -1 + 0.7 = -0.3$ $s_2 = -1 - 0.7 = -1.7$
gives under-critically damped behavior	gives critically damped behavior	gives over-critically damped behavior

2) Note the different start behaviour that depends on the difference in degree of powers in the numerator and denominator polynomials.
3) The Steady-State value depends on the transfer function's limit-value when s approaches zero.

Figure 2. Examples of systematically varied Laplace functions to analyse, mathematically and graphically, in the Transient Response lab.

We have found it to be a critical pre-requisite that students must have experience of variations in different types of solutions and resulting functions. We have therefore developed the set of 6 different transfer functions displayed in figure 2. Basically these consist of permutations of two different numerators and three different denominators. These are first solved by "paper and pencil" and by using Simulink®. In figure 3 we have displayed an analysis of two students' learning in the lab.

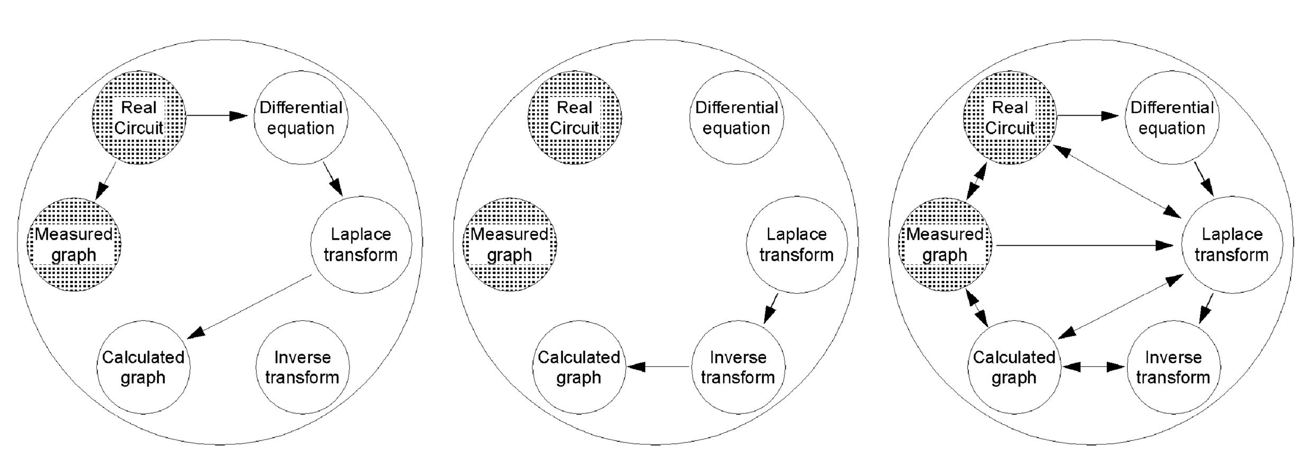


Figure 3. Example of an analysis of learning in the Transient Response lab, using the model for learning a complex concept: a) Student Benny's lived object of learning after the first part of the revised lab: b) Student Tess's lived object of learning at the same time; and c) Student Benny and Tess's lived object of learning at the end of the lab work in the revised lab

In conclusion, our results corroborate variation theory and the necessity of developing tasks that support students' discernment of critical aspects of the object of learning. Our study also underscores Ference Marton's claim that we have to find what is critical for each particular object of learning.

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